A New Approach in Design of Sliding Mode Controller by Optimization State Feedback for Two Wheeled Self Balancing Robot

Ekhlas H. Karam, Rokaia Shalal Habeeb, Noor Mjeed

Abstract—The Two-Wheeled Self-Balancing mobile Robot (TWSBR) is one of the unstable highly nonlinear dynamic systems. This work aims to design a robust controller for controlling TWSBR, in order to solve the balancing and tracking problems. A Sliding Mode Controller based on state feedback (SFSMC) is suggested to solve these problems. In this work, the equivalent –like the term of the SMC’s control law is estimated using a state feedback in order to overcome the dependency of the SMC to the robot model and to reject undesirable effect of interaction toward the improvement of robustness. SFSMC parameters have been tuned using modified Cuckoo Search (MCS) and modified Particle Swarm Optimization (MPSO) algorithms to improve its performance in terms of processing time and response accuracy of the robot system. To measure the performance of the robot system, the Integral Square Error (ISE) has been used as a performance index. Simulation results show improvement in the performance of the TWSBR using SFSMC over the classic SMC in terms of processing time and tracking error.

Keywords—Modified Cuckoo Search (MCS), Modified Particle Swarm Optimization (MPSO), Sliding mode control, Two-wheeled Self-balancing mobile robot.

I. INTRODUCTION

Two-wheeled self-balancing robot (TWSBR) is a highly unstable nonlinear dynamic system. As compared with other mobile robots, TWSBR has great advantages over its small size, simple structure, low cost and flexibility [1][2]. TWSBR is commonly used in different applications such as hospitals, shopping malls’ trolleys and industrial environments [3][4].

Solving the problem of the TWSBR balance and trajectory tracking, have received increasing attention from researchers. To keep the TWSBR in balanced condition, it is required that it stay upright perpendicularly on the ground level [5]. A robust controller that solve the trajectory tracking problem must keep the dynamical system in track, with shortest displacement and driving time [6].

A wide range of controllers are investigated in the literature to tackle the problem of balancing and trajectory tracking of the TWSBR. In an attempt to control the TWSBR conventional control approaches like PID and LQR controller [7][8], nonlinear state feedback controller [9] and the pole placement control [10] as well as modern (optimal /adaptive) controllers have been used to solve the balancing and tracking problem. Modern approaches such as sliding mode controllers and adaptive sliding mode controllers [11][12], adaptive robust regulators and adaptive back-stepping [13][14], optimal Model Predictive Controller (MPC) control for a TWSBR as in [15][16], \( \infty \) controllers [17], PID controllers combined with back-stepping controller [18] are implemented as well.

The Sliding Mode Controller (SMC) is often used to control the TWSBR. Nasir, Ahmad N. K et.al [19], compared the performance of balancing robot under SMC and classical PID controller. While [20]-[22] designed two vigorous SMCs for the position and angle of the self-adjusting robot. Due to the promising results to track the desired trajectory and reject the disturbance when using SMC, a new approach to design SMC is suggested in this work. To enhance the performance of the classical SMC, a state feedback based design (SFSMC) is introduced. A modified Cuckoo Search (MCS) and modified Particle Swarm Optimization (MPSO) algorithms suggested by [23] are used to tune the SMC and SFSMC parameters.

The rest of the paper is organized as follows. Section II presents the dynamic model of the TWSBR. Section III explains the SMC design. Section IV illustrates the MPSO and MCS. Section V presents the SFSMC. Section VI addresses the results of the proposed controller, while the main conclusions are presented in Section VII.

II. TWSBR DYNAMIC MODEL

To design a control unit that steers the robot to the desired location, a dynamic model is required. The mechanical system of the TWSBR is studied as two subsystems separately one for the wheels and the other for the chassis as illustrated in Fig. 1 and Fig. 2, respectively.

Fig.1 Wheels’ free body diagram[24]
Moment of Inertia of the body $290 \times 10^{-3}$ kg.m$^2$

Body Mass $6$ kg

Moment of Inertia of the wheel $1.7 \times 10^{-3}$ kg.m$^2$

Back Electromotive Force Constant $45.8 \times 10^{-3}$ V.s/

rad

The Torque constant $45.8 \times 10^{-3}$ Nm/Amp

Distance between the wheel’s center and the body's center of mass $20$ cm

DC motor Resistance $2.49$ Ω

Wheel Radius $7.7$ cm

Gravity $9.81$ m/s$^2$

Moment of Inertia of the wheel $1.7 \times 10^{-3}$ kg.m$^2$

Moment of Inertia of the body $290 \times 10^{-5}$ kg.m$^2$

Chassis' position m

Chassis’ angle rad

### TABEL I. Physical values of the TWSBR [23]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_p$</td>
<td>Body Mass</td>
<td>6 kg</td>
</tr>
<tr>
<td>$M_W$</td>
<td>Wheel Mass</td>
<td>300 g</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Back Electromotive Force Constant</td>
<td>$45.8 \times 10^{-3}$ V.s/ rad</td>
</tr>
<tr>
<td>$K_m$</td>
<td>The Torque constant</td>
<td>$45.8 \times 10^{-3}$ Nm/Amp</td>
</tr>
<tr>
<td>$l$</td>
<td>Distance between the wheel’s center and the body's center of mass</td>
<td>$20$ cm</td>
</tr>
<tr>
<td>$R$</td>
<td>DC motor Resistance</td>
<td>$2.49$ Ω</td>
</tr>
<tr>
<td>$r$</td>
<td>Wheel Radius</td>
<td>$7.7$ cm</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity</td>
<td>$9.81$ m/s$^2$</td>
</tr>
<tr>
<td>$I_w$</td>
<td>Moment of Inertia of the wheel</td>
<td>$1.7 \times 10^{-3}$ kg.m$^2$</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Moment of Inertia of the body</td>
<td>$290 \times 10^{-5}$ kg.m$^2$</td>
</tr>
<tr>
<td>$x$</td>
<td>Chassis’ position</td>
<td>m</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Chassis’ angle</td>
<td>rad</td>
</tr>
</tbody>
</table>

### III. SLIDING MODE CONTROL

SMC is a robust controller that can handle different nonlinear systems problems [28][29]. The control action consists of two control parts; equivalent (continuous) controller and discontinuous controller, to derive the SMC for TWSBR, (1) and (2) are rewritten as:

$$\ddot{x} = f(x, \theta, t) + b_x u(t) + d(t)$$

(6)

$$\ddot{\theta}_p = f_{\theta_p}(x, \theta, t) + b_{\theta_p} u(t)$$

(7)

Where $f(x, \theta, t)$ and $f_{\theta_p}(x, \theta, t)$ are the unknown nonlinear function of the system position and angle respectively, $u(t) \in R$ is the control action, and $b_x, b_{\theta_p}$ are the control gain for position and angle respectively.

To calculate the sliding mode controller $u_x$ and $u_{\theta_p}$, a sliding surface for position $x$ and angle $\theta_p$ is defined as:

$$s_x(t) = \lambda_x e_x + \dot{e}_x$$

(8.a)

$$s_{\theta_p}(t) = \lambda_{\theta_p} e_{\theta_p} + \dot{e}_{\theta_p}$$

(8.b)

Where

$$e_x = x_d - x$$

(9)

$$e_{\theta_p} = \theta_{pd} - \theta_p$$

(10)

Where $x_d$ is the desired trajectory of position, $\theta_{pd}$ is the desired trajectory of angle, $e_x$ and $e_{\theta_p}$ are the error of position and angle. The values of $\lambda_x, \lambda_{\theta_p}$ are strictly positive and choosing them properly will enhance the SMC performance.

To determine the equivalent control part for $(x, \theta_p)$, sliding surface along the system trajectory is differentiated, then setting it equal to zero as given by:

$$\ddot{s}_x = \lambda_x \dot{e}_x + \ddot{e}_x = 0$$

(11.a)

$$\ddot{s}_{\theta_p} = \lambda_{\theta_p} \dot{e}_{\theta_p} + \ddot{e}_{\theta_p} = 0$$

(11.b)

By choosing the Lyapunov function as:

$$V = \frac{1}{2} e^2 \geq 0$$

(12)
For stability and to obtain invariant and null value sliding variable, it gives the switching condition as:
\[ ss \leq -\eta |s| \]  
(13)
Here, \( \eta \) is a positive fixed parameter. Substituting (11) in (6) and (7), gives the approximation of continuous control law for position (10):
\[ u_{x\text{eq}} = b_x^{-1}[\lambda_x e + \ddot{x}_d - f_x(x, \theta_p)] = b_x^{-1}u_{x\text{eq}} \]  
(14a)
And for angle
\[ u_{\theta\text{eq}} = b_{\theta}^{-1}[\lambda_{\theta} \dot{\theta} + \ddot{\theta}_d - f_{\theta}(x, \theta_p)] = b_{\theta}^{-1}u_{\theta\text{eq}} \]  
(14b)
By replacing (11) into the switching condition (13) and use of (6) and (7) the control law for position and angle is obtained as [11]:
\[ u_x = b_x^{-1} [u_{x\text{eq}} + k_x \text{sign}(s)] \]  
(15)
\[ u_x = u_{x\text{eq}} + u_{xs} \]  
(16)
Here, \( k_x \) is the positive switching gain, \( u_{xs} \) is the discontinuous control, and \( u_{x\text{eq}} \) is the equivalent (continuous) controller provided that \( k_x \) is:
\[ k_x \geq \beta (F + \eta) + (\beta - 1) |u_{x\text{eq}}| \]  
(17)
To decrease the impact of the chattering phenomenon, the boundary layer is considered. The \( \text{sign}(.) \) function is replaced by \( \text{sat}(.) \) function in the boundary layer, so (15) becomes [3]:
\[ u_x = b_x^{-1} [u_{x\text{eq}} + k_x \text{sat}(s)] \]  
(18a)
The same previous step is considered to define the control action \( u_{\theta_p} \) for \( \theta_p \) and final equation will be as follows:
\[ u_{\theta_p} = b_{\theta}^{-1} [u_{\theta_p\text{eq}} + k_{\theta_p} \text{sat}(s)] \]  
(18b)
The sliding mode controllers derived in this section have some parameters that need to be set. These parameters have an effect on the system stability and performance. The parameters \( (\lambda_x, \lambda_{\theta}, k_x \text{ and } k_{\theta}) \) of the SMC are tuned by using the algorithms described in section IV.

expected that the SMC to be robust in the face of the unmodeled uncertainties which is in this approximated dynamics. It is obvious that, in the presence of the large range of uncertainties and disturbances, the performance of SMC will be degraded. In this section, an estimation to the equivalent of each part of the SMCs in TWSBR system by state feedback is introduced to solve the model dependency, not at cost of robustness.

This approach provides a new SMC to attenuate uncertain disturbances for TWSBR with excitation control by using the combination of state feedback with sliding mode control, where feedback gains change the dynamics of a system.

Subsequently the unstable system can be stabilized and the effects of external disturbances can be reduced. Most often, state feedback allows a system to be insensitive to external disturbances

The proposed control law based on state feedback gain will become
\[ u_{\text{eq}} = K_x(t) \]  
(19)
Where \( K = [K_1, K_2, K_3, K_4] \) represent feedback gains and
\[ x(t) = [x, \dot{x}, \theta_p, \dot{\theta}_p] \]  
is the state vector of the robot system.
\[ u_x = u_{x\text{eq}} + u_{xs} \]  
(20)
The SFSMC parameters \( [K] \) are tuned by using the algorithms described in the following section.

V. OPTIMIZATION ALGORITHMS

The population based optimization algorithms or what are known as meta-heuristic algorithms use multiple solutions when exploring the search space to solve an optimization problem. Generally, these algorithms are inspired by some animal’s behavior. In this paper PSO and CS are considered.

A. PSO

PSO algorithm is an optimization technique introduced by Keeney and Ebhart in 1995. PSO mimics the bird flocks or fish schooling behavior. An optimal solution (best solution) can be found through generations’ update [30]. It uses initial random solutions called particles. A population contains \( M \) particles. Each particle has current position \( x_i^t \) and current velocity \( v_i^t \), where
\[ i \] is the particle index (\( 1 \leq i \leq M \)), and \( t \) is the iteration index.
Both \( x_i \) and \( v_i \) are updated using (21 and 22)
\[ v_i^{t+1} = v_i^t + c_1 r_1^t (p_i - x_i^t) + c_2 r_2^t (p_g - x_i^t) \]  
(21)
\[ x_i^{t+1} = x_i^t + v_i^{t+1} \]  
(22)
Where \( c_1 \) and \( c_2 \) are the acceleration coefficients, \( r_1 \) and \( r_2 \) are random numbers of uniform distribution between [0, 1]. \( I_n \) is the inertia weight (\( I_n < 1 \)), \( p_i \) is the best solution of the \( i \)th particle over different iterations and \( p_g \) is the global best solution over all particles and iterations. A problem dependent fitness function \( F(x_i) \) is defined (minimization or maximization) to compare different solutions [31].

An MPSO algorithm is considered here, as introduced in [23] due to its robust performance over standard PSO.

![Fig.3 SMC block diagram for TWSBR](image)
B. CS

Xin-she yang and Suash Deb introduced the CS algorithm in 2009 [32]. The algorithm mimics the cuckoo’s behavior in searching for suitable parasitic nests to lay their eggs. This breeding behavior is idealized by three main rules:

1. Each cuckoo lays one egg at a time and chooses a random host nest to dump the egg in it.
2. High quality nests will carry over to the next generation. Best nests represent solutions with good fitness values.
3. Number of available nests is fixed and the laid egg is discovered by the host at a probability of \( p_r \in [0,1] \). When discovering the laid egg the host bird either get rid of the egg or leaves the nest and build a new one.

In the optimization algorithm the host nest represents a solution to the optimization problem. The location of the host nest represents a fitness value of the algorithm. The search process for a suitable nest maps to the optimization process.

The new suitable nest is generated according to the following law:

\[
\text{nest}_{i+1} = \text{nest}_i + st \odot \text{levy} (\delta)
\]

\( i = 1,2,3, ..., N \)

Where \( \text{nest}_i \) is the \( i \)-th solution at the \( i \)-th generation, \( st \) represents the step size vector that is optimization problem dependent, and \( \odot \) is an entry-wise multiplication. A Levy flight is a random walk in which the steps are defined in terms of the step-lengths that are distributed according to a certain distribution that has an infinite variance and means [32][33].

\[
\text{levy} \approx u = t^{-\delta}
\]

\( t \) in (24) represents a step size drawn from Levy distribution.

In this paper an MCS is considered as introduced in [23] due to its robust performance over standard CS.

VI. SIMULATION RESULT

Matlab software version (R2016a) is considered to illustrate the robustness and efficiency of the suggested controller. Simulations results of linear (step) and nonlinear trajectories with uncertainties are carried out using the suggested control scheme (SF SMC with MPSO, MCS) and compared with classical (SMC with MPSO, MCS). The optimal values of the SF SMC and SMC parameters are obtained using MPSO and MCS optimization algorithms.

Tables 2 and 3 show the MCS and MPSO parameters, while table 4 shows the optimal SMC, SF SMC parameters.

The ISE as described in equation (40) is used as a performance index to check the system efficiency. ISE is used as a fitness function in the MPSO and MCS algorithms as well:

\[
F = ISE = \int_{0}^{\infty} e^2 \, dt
\]

TABLE II. MPSO Parameters.
SFSMC and MPSO in terms of settling time $t_s$ and hitting time $t_h$ as shown in Table 6 and Table 7.

**TABLE VI. Performance parameters using MPSO**

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>$t_s$ (sec.)</th>
<th>$t_h$ (sec.)</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC</td>
<td>4.57</td>
<td>4</td>
<td>0.06</td>
</tr>
<tr>
<td>SFSMC</td>
<td>4.3</td>
<td>2.3</td>
<td>0.041</td>
</tr>
<tr>
<td>$K_{1,2,3,4}=[44,23,1.45,0.1]$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE VII. Performance parameters using MCS**

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>$t_s$ (sec.)</th>
<th>$t_h$ (sec.)</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC</td>
<td>3.8</td>
<td>3.2</td>
<td>0.047</td>
</tr>
<tr>
<td>SFSMC</td>
<td>3.43</td>
<td>2</td>
<td>0.038</td>
</tr>
<tr>
<td>$K=[44,20,1.7,0.1]$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**VII. CONCLUSION**

This work introduces a design to non-linear controllers SFSMC and SMC based on MPSO and MCS optimization algorithms, in an attempt to solve balancing and tracking problems of the TWSBR. The MCS algorithm shows better performance with minimum ISE and fast convergence rate at lower number of iterations, in state feedback SMC. The MCS overcomes the MPSO with respect to different performance parameters.

However, simulation results show that the performance of TWSBR with SFSMC and MCS is more efficient than with SFSMC and MPSO in terms of settling time $t_s$ and hitting time $t_h$. The MCS algorithm achieves better performance with minimum ISE and fast convergence rate at lower number of iterations, in state feedback SMC. The MCS overcomes the MPSO with respect to different performance parameters.
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