Temperature control system synthesis for a thermodynamic system

V.P. Lapshin, A.A. Zakalyuzhnyy

Abstract—**The paper considers a special case of electromechanical control system synthesis by the maximum method. The task of electromechanical system synthesizing the optimal speed for stabilizing the thermodynamic system's operating modes was chosen as the basis, based on the use of the Peltier module as a thermal Converter. An analysis and perturbed motion mathematical model synthesis of the system was performed, which showed the presence of an asymptotically stable stationary operation mode. Then, on the obtained mathematical model basis, the control system was synthesized by the maximum method. Systems modeling was performed using the MATLAB software package. The obtained results allowed us to formulate the following main scientific position: the control system synthesis based on the maximum principle allows us to significantly optimize the reaction of the thermodynamic system to external disturbances. To implement the results obtained, you will need to modify the device we offer as follows: you will need to enter a temperature control sensor and a separate chip for calculating the switching time on, the switching time off and switching of additional control on the fan drive.**

Keywords—**Perturbed motions, optimal control, performance, maximum principle, thermal condition.**

I. INTRODUCTION

In the previous century, the needs of technology development, in particular space technology, became the basis for a new control theory. This control theory is now generalized by the concept of optimal control theory. This direction is based on the method of maximum developed in the 50-60 years of XX century, which was developed by mathematicians L. S. Pontryagin and his students. Since then, many control systems have been built on this approach [1].

Today, taking into account the development of new areas of microelectronics and control systems, new interesting control problems arise, which can involve the developed apparatus of the optimal control theory and, in particular, the maximum method itself. The task of ensuring stabilization of the operating temperature of modern microelectronics is one of these tasks. Given the shrinking size of devices based on semiconductor technology and their increasing performance, the task of removing heat from them becomes one of the most priority [2].

In addition to passive methods for removing heat from heating elements a number of active cooling methods have recently appeared. One of these methods is the method of using the widely known Peltier element. However, there is a new problem associated with the removal of heat from the heating part of this module. We have proposed a new device that allows us to perform this task effectively. The struct of this device is shown in figure 1. Heat removal using this type of element should be as fast as possible, since overheating of microelectronics elements can lead to their failure. The maximum method will be used to developed such a system.

II. SYNTHESIS OF A MATHEMATICAL MODEL OF THE PERTURBED MOTION OF THE SYSTEM

Before the procedure for synthesizing a mathematical model, let's consider a scheme of the installation (figure 1).

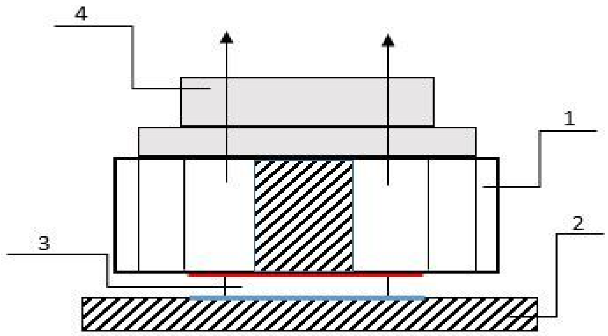

Figure 1. Peltier element with active cooling

In figure 1, the arrows show the direction of heat removal. The system consists of the following elements (figure 1): 1 – radiator, 2 – aluminum bar, 3 – Peltier element, 4 – fan. As a fan drive, we suggest using the widespread 7015-C electric engine. The mathematical model of a DC motor with collector control is described by the following system of equations [3]:

$$U - c_e\omega = L\frac{di}{dt} + Ri, \qquad (1)$$
$$c_m i = J\frac{d\omega}{dt} + M_c$$

where U- voltage;
i – amperage
R, L – parameters of the electric part of the engine;
J – moment of inertia;
ω – motor rotor rotation frequency;
Mc= $\mu\omega^2$ - external applied resistance moment, for the case in question $\mu = 1.1 \cdot 10^{-7}$;

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c_m, c_e – mechanical and electrical stability of the engine.
The thermodynamic parameters of the device can be described by a differential equation:

$$T\frac{d\theta}{dt}+\theta=N_1-N_2, \quad (2)$$

where θ – current temperature;
N_1 – source power, in this case 223. 58 Joule;
N_2 – drain power, in this case it is a function of the angular velocity: $N_2 = f(\omega)$, T - time constant of the Peltier element, obtained empirically and equal to 2.85 seconds.
Taking into account the aperiodic nature of heat propagation, we assume the structure of the functional dependence of the flow capacity in the form $f(\omega) = N_1(1-e^{-\alpha\omega})$, then $N_2 = N_1(1-e^{-\alpha\omega})$. Based on this, it is fair to convert equation (2) to the form:

$$\frac{d\theta}{dt}=\frac{N_1}{T}e^{-\alpha\omega}-\frac{\theta}{T} \quad (3)$$

Then the system of equations describing the thermodynamics of the Peltier element cooling will be:

$$\begin{aligned}\frac{d\theta}{dt}&=\frac{N_1}{T}e^{-\alpha\omega}-\frac{\theta}{T}\\ \frac{di}{dt}&=\frac{U}{L}-\frac{c_e\omega}{L}-\frac{Ri}{L}\\ \frac{d\omega}{dt}&=\frac{c_m i}{J}-\frac{M_c}{J}\end{aligned} \quad (4)$$

After replacing the variables and introducing generalized model parameters, we get the model in new coordinates:

$$\begin{aligned}\frac{dx_1}{dt}&=-a_{11}x_1+a_{12}e^{-\alpha x_2}\\ \frac{dx_2}{dt}&=-a_{22}x_2^2+a_{23}x_3\\ \frac{dx_3}{dt}&=-a_{32}x_2-a_{33}x_3+bU\end{aligned} \quad (5)$$

The results of modeling the system of equations (5) showed that the system has a stable mode of operation. When applying a power supply voltage of 12 V. to the motor, a stable temperature regime is observed in the cooled part of the Peltier element (figure 1). The temperature in this part is stabilized in the vicinity of the value of 62 °C, which meets the requirements for such cooling systems. However, such a system must not only provide a stable temperature regime, but also the required response time to external perturbations. To estimate the reaction time of the system to external disturbances, we transform the system of equations (5) into a system of equations in the coordinates of the perturbed motion of the system. This transformation is acceptable from the point of view of modeling the dynamics of the system [4-7]. To move from the original model of the system represented by equations (5) to the model of perturbed motion, we replace the coordinates of the system state as follows:

$$\begin{aligned}x_1&=x_{01}+z_1\\ x_2&=x_{02}+z_2\\ x_3&=x_{03}+z_2\end{aligned}, \quad (6)$$

where x_{01}, x_{02}, x_{03} - constants that characterize the equilibrium stationary mode of operation of the system,
z_1, z_2, z_3- coordinates of the perturbed motion of the system.

Taking into account (6) and after subtracting from the resulting system of equations the system of equations describing the stationary state, we obtain the following system of equations for the perturbed motion of the system:

$$\begin{aligned}\frac{dz_1}{dt}&=-a_{11}z_1+a_{12}e^{-\alpha z_2}e^{-\alpha x_{02}}-a_{12}e^{-\alpha x_{02}}\\ \frac{dz_2}{dt}&=-a_{22}z_2^2-2a_{22}z_2x_{02}+a_{23}z_3\\ \frac{dz_3}{dt}&=-a_{32}z_2-a_{33}z_3\end{aligned}. \quad (7)$$

The results of modeling the control system (7) for the case of a temperature jump in 5 °C of the heating part of the Peltier element are shown in figure 2.

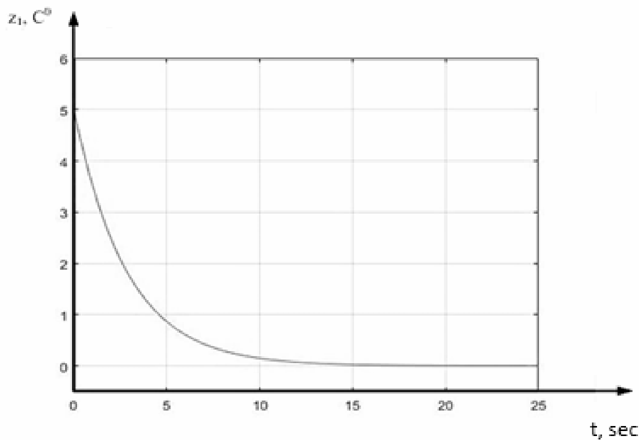

Figure 2. Particular case of perturbed motion time characteristic in the control system.

As can be seen from figure 2 the control system is characterized by the asymptotically stable stationary operation mode presence. For this case, the reaction time to an external disturbance was about 15 seconds. In this case the zero temperature in figure 2 corresponds to 62 °C for the system case in the original coordinates. To speed up the system's response to external disturbances, we use the maximum principle and synthesize the optimal control for the fan drive. The following restriction will be applied to control: -2≤ΔU≤2 B, in this case, the engine voltage is 12 V.

III. Synthesis of control based on the maximum principle

The first step is to get a linearized model of perturbed motion [8]. A linearized system of equations describing perturbed motion looks like this:

$$\begin{aligned}y_1&=-\beta_{11}y_1-\beta_{21}y_2\\ y_2&=-\beta_{22}y_2+\beta_{23}y_3\\ x_3&=-\beta_{32}y_2-\beta_{33}y_3+b\Delta U\end{aligned}, \quad (8)$$

where y_1, y_2, y_3 - the linearized coordinates of the disturbed motion,

$a_{11}=\beta_{11}$,

$\alpha a_{12}e^{-\alpha x_{02}}=\beta_{12}$,

$2a_{22}x_{02}=\beta_{22}$,

$a_{23}=\beta_{23}$,

$a_{32} = \beta_{32}$,

$a_{33} = \beta_{33}$.

For the case of system (8), we define the Hamiltonian

$$H = \Psi_1 f_1 + \Psi_2 f_2 + \Psi_3 f_3 \quad (9)$$

where ψ_i – undefined coordinates, f_i – right parts of expressions in the system (8). Let's make a conjugate system of equations for auxiliary variables ψi, which in our case will take the following form:

$$\begin{cases} \frac{d\psi_1}{dt} = \beta_{11}\psi_1 \\ \frac{d\psi_2}{dt} = \beta_{12}\psi_1 + \beta_{22}\psi_2 + \beta_{32}\psi_3 \\ \frac{d\psi_3}{dt} = -\beta_{23}\psi_2 + \beta_{33}\psi_3 \end{cases} . \quad (10)$$

Solving a system of linear equations (10) for our drive case, where $\beta_{11} = 0.35$,

$\beta_{12} = 0.27$,

$\beta_{22} = 1.69$,

$\beta_{23} = 5487$,

$\beta_{32} = 20.83$,

$\beta_{33} = 16583.3$,

$b = 206.74$, takes the form:

$$\begin{cases} \psi_1 = 0.9994 C_3 e^{0.35t} \\ \psi_2 = 0.0013 C_1 e^{16576t} + 0.9493 C_2 e^{8.59t} - 0.0328 C_3 e^{0.35t} \\ \psi_3 = C_1 e^{16576t} + 0.3143 C_2 e^{8.59t} - 0.0108 C_3 e^{0.35t} \end{cases} , \quad (11)$$

where C_1, C_2, C_3 – the constant of integration.

After substituting the equations system (11) in (9), we get the part of the Hamiltonian that includes the control:

$$H^* = \psi_3 U = (C_1 e^{16576t} + 0.3143 C_2 e^{8.59t} - 0.108 C_3 e^{0.35t}) bU . \quad (12)$$

Analysis of the expression (12) shows that the Hamiltonian will tend to the maximum if the control sign is the same as the function sign Ψ_3. That is, in order for the Hamiltonian H to take the maximum positive value, the term H* must always be positive and the largest. To do this, the optimal control algorithm must have the form $u(t) = \sigma Umax$, где $\sigma = sign(\Psi_3)$. Let's build it using the partial values of the integration constants (figure 3).

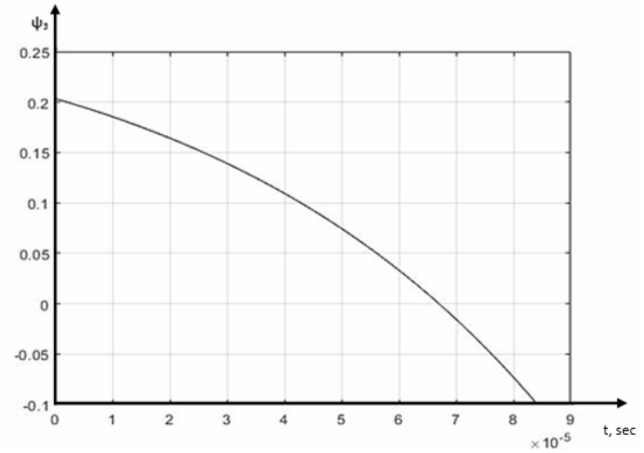

Figure 3. Time characteristic of the function Ψ_3

Analysis of figure 3 allows us to conclude that the control is a piecewise constant function that takes values of ±2 V, it will have no more than two constancy intervals, since the nonlinear function ψ_3 changes the sign no more than once. In this case, the sign changes from plus to minus, that is, to fulfill the maximum principle, it is first necessary to apply U = +2 V to the engine, and then U = -2 V.

Based on the above, we define an algorithm for the optimal equation that provides for the transfer of an object from the missing state $x_1(0)=5$, $x_2(0)=0$, $x_3(0)=0$ to the original state $x_1(T)=0$, $x_2(T)=0$, $x_3(T)=0$ in the minimum time. It should be noted that in modern conditions there is no need to obtain an analytical the original equations system solution. Using the Matlab, you can easily and visually get a numerical solution for the case in question. The results of numerical simulation for the calculated optimal control algorithm are presented in figure 4.

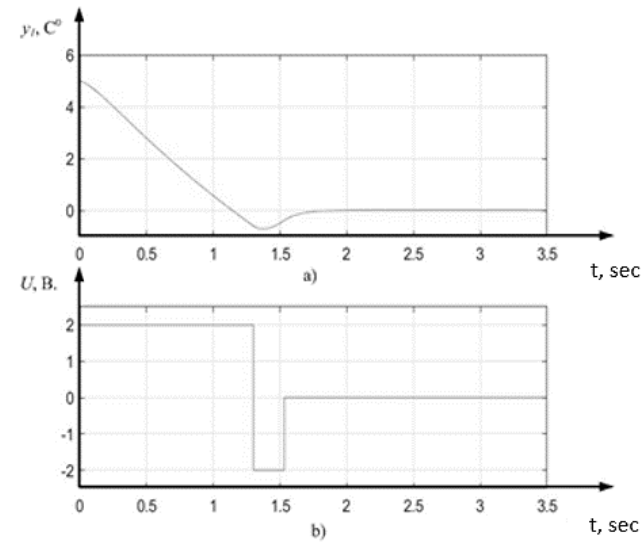

Figure 4. Time characteristic of a particular case of perturbed motion in the control system a) and synthesized control b)

The analysis of the characteristics presented in figure 4 shows that the control time for the considered control case, which is optimal in terms of speed, was about 1.5 seconds. Comparing these data with the results shown in figure 2, we can conclude that the control system performance has increased tenfold.

IV. Conclusion

The Hamiltonian (9) synthesized in this paper allowed to increase the system response rate to external disturbances by almost ten times, which in turn allows us to talk about more reliable cooling of electronic blocks and devices using control built on the principle of maximum. We can formulate the following scientific proposition: the synthesis of the control system based on the maximum principle allows us to optimize the reaction of the thermodynamic system to external disturbances.

To implement the results obtained, you will need to convert the device we offer (figure 1). To do this, you will need to enter a temperature control sensor and a chip to calculate the time to turn on, off, and switch additional controls on the fan engine.

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