Mathematical modelling of the dynamics of AIDS epidemics development in the world

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Abstract— **In this work, mathematical modelling of the dynamics of AIDS epidemics development in the world is performed for the following countries: Russian Federation, Austria, France and Brazil. HIV infection and AIDS are the diseases for which the development is under special control of international community. Annual HIV infection morbidity is up to 3 million cases a year, and the average age of the infected varies over the range of 20-30 years. At the moment, there is no cure for HIV infection. Without supporting treatment, it leads to the development of acquired immune deficiency syndrome (AIDS) and death. Consequently, the spreading of HIV leads to serious economic loss. Mathematical methods are widely applied to the analysis of HIV spreading, forecasting the epidemic development and qualitative evaluation of measure effectiveness. Mathematical description of the HIV epidemic development process allows analyzing the character of the disease for each region deeper, perform the comparative study of statistical and modelled data, make suggestions on further possibilities of HIV infection development and confirm the importance of introducing federal measures of confronting human immunodeficiency viruses. Therefore, authors provide the results of analyzing statistical data for the counties mentioned above, mathematical models of AIDS epidemic development and results of modelling for each region. Over the course of this work, optimal parameter values have been found for an accurate description of statistical data. A comparative study of the current situation for AIDS epidemic in Russian Federation, Austria, France and Brazil. As a result, the conclusion of effectiveness of present measures of confronting HIV infection in these countries.**

Keywords— **HIV infection; AIDS; mathematical modelling; epidemiological modelling; SIR model.**

I. Introduction

Despite the rapid development of science and technology, many unsolved problems still remain in the modern world. One of such problems is the fight against human immunodeficiency virus (HIV). HIV is one of the most dangerous and quickly spreading diseases in the world.

First cases of the HIV disease were reported at the end of 1970s. Only in 1982, after a thorough study, the disease received its name - acquired immune deficiency syndrome (AIDS). Then, AIDS was called the “4H” disease, what referred to heroin users, homosexuals, hemophiliacs, and Haitians. Since that time, 78 million people have been infected with HIV, 35 million people have died from AIDS [1]. There’s no cure for HIV. What’s more, without constant therapy it leads to the development of acquired immune deficiency syndrome (AIDS) and, at last, to death. So, the increase in the number of the infected can lead to serious economic and demographic loss [2].

Every year new measures of confronting AIDS are developed: scientific research is conducted and statistical data are analyzed. In most cases, the forecasts of leading organizations that develop programs for fighting the HIV virus are based on evaluations obtained with the use of mathematical models [3,4]. Consequently, constructing mathematical models is a very important stage in the disease resistance process. Results obtained in this work can be used for the development of preventive measures of such kind.

Mathematical models published so far solve the following problems connected with the problem of controlling the spreading of the HIV infection: investigation immunologic dynamics for the case of one organism, forecasting the development of epidemic at the population level and estimating the consequences in economics and social sphere [5-10].

Solution of the forecasting the consequences of the HIV infection problem for economic and social spheres is one of the most important tasks in the field of managerial decision-making. As a rule, specialists choose to correct for already existing models according to expert evidence on the HIV/AIDS epidemic influence [11,12].

The task of modelling immunological dynamics of the HIV infection in terms of one organism is aimed at studying biological processes of virus interaction with its host. Appearance and development of immunological models is connected with the works of such researches as Perelson and Nelson, Novak and May, Smith and Wall, Commenges and Joly, Culshow and others.

Epidemiologic dynamics models for the HIV infection first appeared in 1980s almost at the same time as the virus was discovered. Epidemiological models can be classified in three ways: populational (individual-oriented), determined (stochastic) and analytical (imitational). Currently, populational approach is the most popular one. It is based on reducing the investigation of a community of individuals to considering classes of individual relation to the infection (susceptible, infected, immune) [13].

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II. MODELLING THE DYNAMICS OF AIDS EPIDEMIC DEVELOPMENT IN THE WORLD

In this paper, mathematical model for the AIDS epidemic development based on statistical data of the number of the infected and the dead [14,15]. Let N be the country's population (population increase due to birth rate not taken into account), G is the population of the risk group (i. e. the number of healthy people that may get infected), I is the number of the infected, E is the number of "eliminated", i. e. dead, and also isolated from the others. Morbidity is proportional to the number of contacts between people of the risk group G and the infected I. Rate at which people leave the infected group I, i.e. die, is proportional to the population of this group. Consequently, the development of AIDS epidemic can be described by the following system of differential equations:

$$\begin{cases} \dfrac{dG}{dt} = -aGI, \\ \dfrac{dI}{dt} = aGI - bI, \\ \dfrac{dE}{dt} = bI, \end{cases} \qquad (1)$$

where a, b are the parameters.

By setting the values of a and b and the initial values of G and I, one can calculate the dynamics of the disease [16-18].

An accurate approximation of statistical data was obtained because the interaction between infected and healthy in the certain region takes place only within the "risk group". The size of this group depends on particular characteristics of a region and serves as a representation of the region's culture and habits. It can be explained by the fact that AIDS epidemic has not affected the whole world yet. The interaction between people from the risk group and the others, what leads to the spreading of the disease, is still low. The differential equations system with initial conditions is solved using numerical methods.

A. Russian Federation

Statistical data on the dynamics of AIDS spreading in Russia are taken from [UNAIDS]. By the end of 2017, over 1 million if HIV-infected were registered in Russia, and the national HIV index for adults equals to 0,7% [19]. Over the course of development of this model in Russian Federation could sustain the demographical decrease in the country. AIDS epidemic in Russia is connected with the factors originating from economic and political difficulties with social background of the 1990s. In the considered model, the size of the risk group is 10 million people, $a = 0.1452$, $b = 0.025$ (Fig. 1). An accurate correspondence of model data to the statistical data can be seen. This shows the high model adequacy for this country (influence of measures taken for restricting AID epidemic is not taken into account). On the basis of modelled curves, one can forecast that the number of people dying from AIDS could surpass 7 million people by 2090, unless serious measures of confronting AIDS are taken.

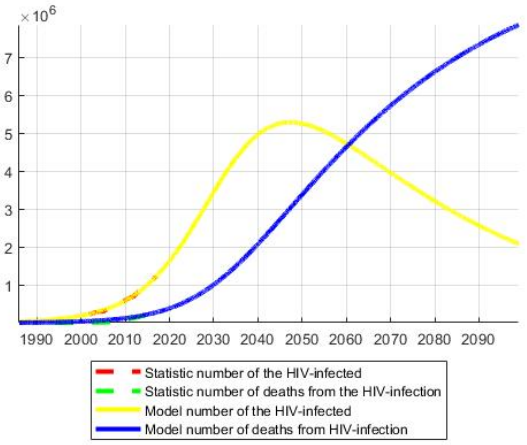

Fig. 1. Dynamics of AIDS development in Russian Federation.

B. Austria

The number of people infected by AIDS in Austria is relatively low in comparison with other European countries. However, there can be no talk of the absence of AIDS epidemic. Around 0,01% of population is infected with HIV [UNAIDS]. Most probably, the virus was brought to this country by immigrants or tourists [20]. In the considered model, the size of the risk group is 10,5 thousand people, $a = 0.1162$, $b = 0.013$ (Fig. 2). A good correspondence of modelled data to the statistical data on HIV infection can be observed. The dynamics of the number of people dying from AIDS can be explained by the national policy aimed at the fight against this virus.

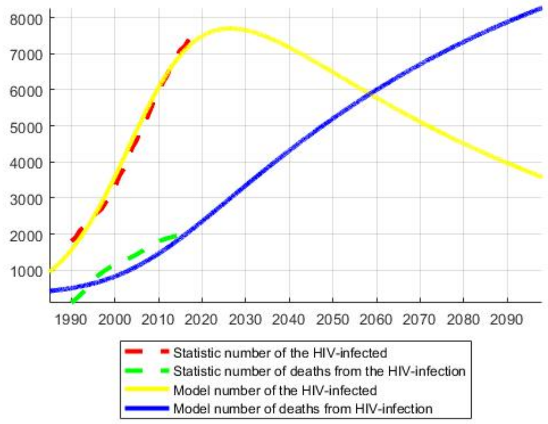

Fig. 2. Dynamics of AIDS spreading in Austria.

C. France

Despite scientific development, the topic of HIV and AIDS still remains a taboo. People living with HIV are afraid of talking about their status and are prone to discrimination. HIV epidemic affects such social groups as prisoners, drug addicts and migrants, etc., more and more often. According to research, 58% of migrants get HIV during the first six years of staying in France. 42% are infected after six years of staying. Statistical data on HIV disease in France are taken from [UNAIDS]. Therefore, the size of the risk group is 280 thousand people, $a = 0.1162$, $b = 0.01$ (Fig. 3). A good correspondence of modelled data curve to the statistical data curve can be observed. As well as for Austria, the dynamics of the number of people dying from AIDS in France can be explained by the national policy aimed at the fight against this virus.

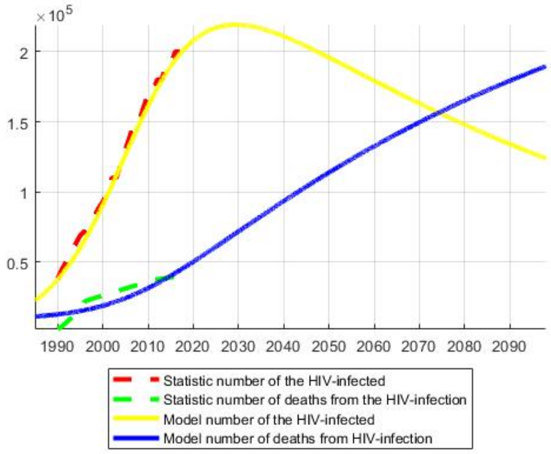

Fig. 3. Dynamics of AIDS spreading in France

D. Brazil

In due course, Brazil encountered a large epidemic of HIV infection. At the beginning of the 1990th forecasts said that over a million people infected by HIV would live in Brazil by 2000. However, due to administrative policy, this forecast didn't turn into reality. Moreover, Brazil has become one of the first country among developing ones to provide life-saving HAART therapy to the patients. As discovered later, this therapy prevents new infection cases. Due to the measures taken, the epidemic is not growing and is concentrated only in crucial social groups (women). Traditionally, there is cultural dominance of men over women. Even though women have the same rights as men, abortions are prohibited in the country, and the laws against domestic abuse are quite weak. That's why mostly poor social classes and negroid population are infected by AIDS. Index of HIV epidemic spreading is at the level of 0,61% [21]. As a result, the spreading of AIDS epidemic in Brazil can be described by the following system of differential equations:

$$\begin{cases} \frac{dG}{dt} = -a, \\ \frac{dI}{dt} = a - b, \\ \frac{dE}{dt} = b, \end{cases} \qquad (3)$$

where a, b are the parameters.

One can calculate the dynamics of disease spreading in Brazil by numerically solving the system 3. Authors have obtained that the size of the risk group is 3 million people $a = 0.132,\ b = 0.0046$ (Fig. 4).

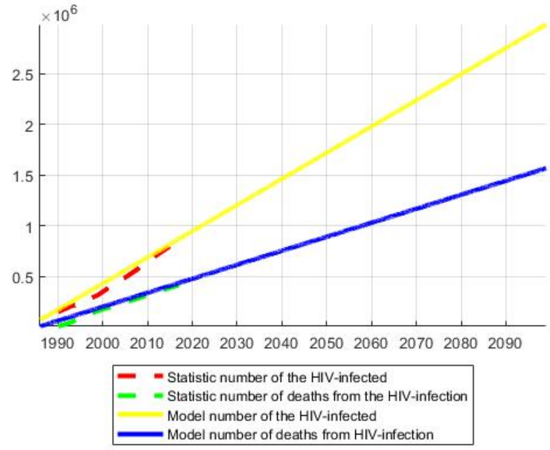

Fig. 4. Dynamics of AIDS spreading in Brazil

III. Conclusion

AIDS is a probable pandemia of the 21st century [22]. The development of mathematical models that allow explaining observed differences in the spreading and morbidity of the HIV infection in the world and forecasting the spreading of AIDS development in various countries is relevant as never before [23].

In this work, the mathematical model of AIDS epidemic development in the world is provided. Modelled curves were built on its basis for the following countries: Russia, Austria, France and Brazil. In all cases, a good correspondence of modelled data to the statistical data curve can be observed, and the dynamics of the number of people dying from AIDS can be explained by the national policy aimed at the fight against this virus in different countries.

Russia is a world leader in the incidence of HIV to date. Despite the measures that are taken to confront AIDS the loss of life among HIV-infected is growing. In 2017, HIV virus caused more than half of deaths from infectious diseases (57%, Rosstat). Increased mortality can be attributed to the lack of observation and treatment of HIV-infected in AIDS-centers. However, one cannot but conclude the measures aimed at combating AIDS. Therefore, there is a clear bias towards the promotion of family values and fidelity. The number of people receiving anti-HIV therapy is increasing annually, but this indicator is far from the benchmark figure of 90-90-90 (UN members pledged to identify 90% of HIV-infected by 2020, of which 90% take an anti-HIV therapy and 90% suppress the HIV virus. The obtained curves (fig. 1) can be considered confirmation of this conclusion, and the forecast, based on these results, is lamentable - in the absence of appropriate measures, by 2090 the loss of life from the AIDS epidemic will reach 7 million people.

Although the spread of HIV can be stopped by effective public health measures, the number of HIV-incidents in Europe is still high [24]. Over the past thirty years, more than 2.3 million incidents of HIV-infections have been identified and recorded in the European Region. Too many people throughout the European Region are diagnosed as HIV-positive at a late stage (53%), which increases the risk of developing diseases, lethality and further spread of HIV [25]. Every fifth HIV-infected person does not know about his infection. Even such developed countries as Austria and

France have not been spared from AIDS pandemic. In Austria, tourists and migrants largely contributed to the spread of the disease. The risk group is probably less in this case, but one cannot speak about the absence of the epidemic (fig. 2). Nevertheless, measures aimed at combating HIV infection at the national policy level are effective, allowing the control of the loss of life. The main population group vulnerable to HIV-infection in France are drug addicts, migrants, prisoners etc. As in Austria, the government is interested in reducing the number of people with AIDS, which is reflected in the model curves (fig. 3). The statistical curve for the number of deaths is below the model one, which indicates the effectiveness of current measures.

The history of the AIDS epidemic update in Brazil is mixed. Thanks to the policy undertaken by the government, the State managed to stabilize the spread of the HIV epidemic, which has been at a level of 0,61% since 2000. Treatment programs include an increase in medical facilities, in which services are provided to all in need for free. Brazil is a clear example of how to confront the epidemic. Nowadays, the epidemic is not growing and is concentrated only in certain groups of the population - drug addicts, people engaged in prostitution, residents of poor areas, migrants. In this regard the authors have changed the system of differential equations (1) to (2).

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Apendix

Statistical data sources:

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