Quasi-optimal Quaternion Genetic Algorithm for Reorientation of the Spacecraft Orbit

Ilia A. Pankratov

***Abstract* — The problem of optimal reorientation of the spacecraft orbit is considered in quaternion formulation. Control (acceleration from jet thrust vector orthogonal to the plane of the orbit) is limited in magnitude. It is necessary to minimize the energy costs for the process of reorientation of the spacecraft orbit. The actual special case of the problem, when the spacecraft's orbit is circular and control is constant on adjacent parts of active spacecraft motion was considered. We have to determine the lengths of the sections of the spacecraft motion and the magnitude of control on each section. Original genetic algorithm for finding the trajectories of spacecraft optimal flights is built. Examples of numerical solution of the problem for the case when the difference between the initial and final orientations of the spacecraft's orbit is equal to a few degrees in angular measure are given. Specific features and regularities of the process of optimal reorientation of the spacecraft orbit are established.**

***Keywords* — gene, optimal control, quaternion, spacecraft.**

I. Introduction

In this paper we consider the problem of optimal reorientation of a circular spacecraft orbit regarded as a unchangeable figure in the course of motion control. It is known that the orbits of the satellite groups GLONASS and GPS are close to circular. The motion of a spacecraft, which is considered as a material point of a variable mass, is studied in the orbital coordinate system with an origin at the spacecraft center of mass. It is required to determine the optimal control $\boldsymbol{u}$ (vector of jet acceleration) which transfers spacecraft from its initial orbit to desired one. Also we have to minimize the energy consumption for this reorientation.

It is well known that the problem of spacecraft interorbital flights is greatly simplified if the start and final orbits lie in the same plane. It becomes possible to find the optimal transition trajectories analytically (accurately or approximately). This has led to the significant number of publications in this area. Note also that due to its complexity, the problem of performance was rarely solved (we can note papers [1]–[4]). Basically the energy cost or the characteristic velocity was minimized (refer to the papers of I.S. Grigoriev, K.G. Grigoriev [5]–[8], S.N. Kirpichnikov and coauthors [9], [10]).

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I. A. Pankratov is with the Saratov State University and Institute of Precision Mechanics and Control Problems of the Russian Academy of Sciences, Saratov, Russian Federation (phone: 8-8452-518480; e-mail: PankratovIA@info.sgu.ru).

In these papers optimal control problems were solved on the basis of the maximum principle. Boundary value problems of the maximum principle were solved numerically by shooting method. In the present article we propose a genetic algorithm to solve the problem of reorientation of the spacecraft orbit.

II. Statement of the Problem

It is required to transfer spacecraft whose motion on the circular orbit is described by equations [11]

$$2\frac{d\boldsymbol{\lambda}}{dt} = \boldsymbol{\lambda} \circ \boldsymbol{\omega}_\eta, \quad \boldsymbol{\omega}_\eta = \frac{ur}{c}\boldsymbol{i}_1 + \frac{c}{r^2}\boldsymbol{i}_3,$$

$$r = \text{const}, \qquad c = \text{const}, \qquad \frac{d\varphi}{dt} = \frac{c}{r^2}$$

from specified initial state

$$t = 0, \quad \varphi(0) = \varphi_0,$$

$$\boldsymbol{\lambda}(0) = \boldsymbol{\lambda}^0 = \boldsymbol{\Lambda}^0 \circ \left(\cos\frac{\varphi_0}{2} + \boldsymbol{i}_3 \sin\frac{\varphi_0}{2}\right) \tag{1}$$

into the final state

$$t = t^* = ?, \quad \varphi(t^*) = \varphi^*,$$

$$\text{vect}\left[\overline{\boldsymbol{\lambda}}(t^*) \circ \boldsymbol{\Lambda}^* \circ \left(\cos\frac{\varphi^*}{2} + \boldsymbol{i}_3 \sin\frac{\varphi^*}{2}\right)\right] = \mathbf{0} \tag{2}$$

with the bounded (in magnitude) piecewise constant control ($|u(t)| \le u_{\max}$)

$$u(t) = \begin{cases} u_1, & \text{if} \quad 0 \le t < \Delta_1, \\ u_2, & \text{if} \quad \Delta_1 \le t < \Delta_1 + \Delta_2, \\ \dots \\ u_M, & \text{if} \quad \sum_{k=1}^{M-1} \Delta_k \le t \le \sum_{k=1}^{M} \Delta_k = t^*. \end{cases}$$

Also we have to minimize the functional

$$J = \int_0^{t^*} u^2 dt = \sum_{k=1}^{M} u_k^2 \Delta_k. \tag{3}$$

Here $\boldsymbol{\lambda}$ is the quaternion of orientation of the orbital system of coordinates, u is the projection of control $\boldsymbol{u}$ on the axis orthogonal to the orbital plane, r is the radius of the spacecraft orbit, c is the modulo of the spacecraft velocity moment, $\boldsymbol{i}_k, k = \overline{1,3}$ are the unit vectors of a hypercomplex space (Hamilton imaginary units), φ is the true anomaly, $\overline{\boldsymbol{\lambda}}$ is the conjugate quaternion, "$\circ$" is the symbol of quaternion multiplication, $\boldsymbol{\Lambda}$ is the quaternion of orientation of the spacecraft orbit.

Note that the quantity of active motion parts M is assumed to be given, but the final time t^* is not fixed.

Functional (3) characterizes the energy consumption for a spacecraft transfer from the initial to final state. We have to determine the value of control u_k on each part of the spacecraft active motion and durations Δ_k of those parts.

It is known that similar problems were considered earlier by S.A Ishkov and V.A Romanenko [12]; O.M. Kamel, B.E. Mabsout and A.S. Soliman [13], [14]; A. Miele and T. Wang [15]; S.Yu. Ryzhov and I.S. Grigoriev [16]. Unfortunately, most authors were deal with equations in angular elements (or Cartesian coordinates). Also they were often studied only transfers between coplanar or closed to each other orbits.

III. Numerical Algorithm

To obtain numerical solution, the equations and relations of the boundary optimization problem were written in the dimensionless form. The dimensionless variables and control are connected with dimension analogues by the relations: $r = Rr^{dl}$, $t = Tt^{dl}$, $u = u_{\max} u^{dl}$. Here R is a typical distance (radius of the initial orbit of the controlled spacecraft), V is a typical velocity, C is a typical sector velocity, and T is a typical time, determined as $V = \sqrt{fM_{attr} / R}$, $C = RV$ and $T = R / V$, respectively. Here f is the gravitational constant, M_{attr} is the mass of attracting body (Earth). Note that in the transition to dimensionless variables in the equations the typical dimensionless parameter $N^b = u_{\max} R^3 / C^2$ arises.

Let us present the equations and relations of the optimization boundary value problem in the dimensionless variables (superscripts “dl” are omitted). The equations of the motion of the spacecraft center of mass take the form

$$2\frac{d\boldsymbol{\lambda}}{dt} = \boldsymbol{\lambda} \circ \boldsymbol{\omega}_\eta, \quad \boldsymbol{\omega}_\eta = N^b u \boldsymbol{i}_1 + \boldsymbol{i}_3,$$

$$r = 1, \qquad \frac{d\varphi}{dt} = 1.$$

The dimensionless optimal control is subject to condition $-1 \le u \le 1$.

Earlier in papers [17], [18], the posed problem was solved with the help of the L.S. Pontryagin maximum principle [19]. As a result of the maximum principle application, a boundary value problem with a movable right end was obtained. It was solved numerically using the shooting method [20]. Unfortunately, there are no formulas for finding unknown initial values of conjugate variables in this problem. The initial approximations for the values of conjugate variables do not converge well to those values that deliver zeros to the residual functions. Iterative methods constantly fall into local minima of residual functions. In this paper genetic algorithm for solving this problem is constructed. Using this algorithm, one does not need to search for the initial values of the conjugate variables. Such methods based on artificial intelligence and machine learning were considered, for example, in [21], [22]. Let us describe the main stages of the proposed genetic algorithm, following the book [23].

First of all, one needs to randomly generate a population of $N_{\max}$ candidate solutions (individuals or phenotypes). Each of them is a set of M pairs of real numbers. In this paper, instead of real numbers Δ_k and u_k, we store in the memory pair of integer numbers $\left(\Delta_k^{\text{int}}, u_k^{\text{int}}\right)$ (gene). The relationship between the desired real numbers and the gene is given by the formula

$$\left(\Delta_k, u_k\right) = \left(\frac{\Delta_{\max} \cdot \Delta_k^{\text{int}}}{2^L - 1}, -1 + \frac{2 \cdot u_k^{\text{int}}}{2^L - 1}\right)$$

The integer numbers Δ_k^{int}, u_k^{int} are from interval $\left[0; 2^L - 1\right]$.

At the second step of the algorithm for each individual one should find the quaternion of orbital coordinate system orientation at $t = t^*$ by the formula [24]:

$$\boldsymbol{\lambda}\left(t_k\right) = \boldsymbol{\lambda}\left(t_{k-1}\right) \circ \left(\cos\frac{\omega\Delta_k}{2} + \frac{\boldsymbol{\omega}_\eta}{\omega}\sin\frac{\omega\Delta_k}{2}\right),$$

$$t_k = \sum_{j=1}^{k} \Delta_j, \quad \boldsymbol{\omega}_\eta = N^b u_k \boldsymbol{i}_1 + \boldsymbol{i}_3,$$

$$\omega = \left|\boldsymbol{\omega}_\eta\right| = \sqrt{1 + \left(N^b u_k\right)^2} = \text{const}$$

with initial conditions (1) (control corresponds to chosen gene).

Fitness function is given by the formula

$$err(t) = \left|\text{vect}\left[\overline{\boldsymbol{\lambda}}(t) \circ \boldsymbol{\Lambda}^* \circ \left(\cos\frac{\varphi}{2} + \boldsymbol{i}_3 \sin\frac{\varphi}{2}\right)\right]\right|. \quad (4)$$

The fitness function (4) describes accuracy of fulfillment of conditions (2) on the right end of the spacecraft trajectory. We compute the value of the fitness function at the point $t = t^*$.

It is well known that natural selection is the process whereby organisms better adapted to their environment tend to survive and produce more offspring. In our case, the smaller value of the fitness function corresponds to the more adapted individual, i.e. the candidate solution used as the argument of the fitness function. If at this step the value of the fitness function for some individual is less than the given small number ε then the algorithm ends, and the control corresponding to this individual is the solution of the problem. If the maximum number of iterations $N_{\max}^{iter}$ is exceeded, the control corresponding to the individual with the minimum value of the fitness function is a solution to the problem.

At the third step of the algorithm we should discard half of the individuals with the highest (worst) values of the fitness function (the number of individuals should be even). Then the crossing of the individual with the lowest value of the objective function with all the others, including itself, is performed. The method of intermediate recombination was chosen as the crossover operator [23]. Children are created

according to the following rule:
$$\text{Child} = \text{Parent1} + \alpha \cdot (\text{Parent2} - \text{Parent1}).$$

Here α is a floating-point pseudo-random number in the range of -0.25 up to 1.25.

A separate multiplier α is selected for each gene of the child. Both numbers in the resulting child genes are rounded to the nearest integers lying on the interval $[0; 2^L - 1]$. At the end of this step we will create a new population of $N_{\max}$ individuals.

At the last step of the algorithm we calculate the average value of the fitness function for the population obtained in the third step. If it is greater than the average value of the fitness function, calculated at the second step, then individuals in the population mutate. To do this, we write genes of all individuals in binary form (for both numbers in each gene, exactly L bits are allocated) and we invert of a randomly selected bit of in these numbers with probability $p_{mut} \in (0;1]$. Then we return to the second step of the algorithm.

Note that the described algorithm should be used repeatedly for different initial populations. In this case, several solutions will be obtained, from which it is necessary to choose the one that corresponds to the reorientation of the spacecraft orbit with less energy consumption.

IV. An Example of Numerical Solution

The components of the quaternion $\boldsymbol{\lambda}$ and $\boldsymbol{\Lambda}$ can be expressed through angular elements of an orbit (the longitude of the ascending node Ω_u, the orbit inclination I, the pericenter angular distance ω_π) and the true anomaly:

$$\boldsymbol{\Lambda} = \left(\cos\frac{\Omega_u}{2} + \boldsymbol{i}_3 \sin\frac{\Omega_u}{2}\right) \circ \left(\cos\frac{I}{2} + \boldsymbol{i}_1 \sin\frac{I}{2}\right) \circ$$
$$\circ \left(\cos\frac{\omega_\pi}{2} + \boldsymbol{i}_3 \sin\frac{\omega_\pi}{2}\right),$$
$$\boldsymbol{\lambda} = \boldsymbol{\Lambda} \circ \left(\cos\frac{\varphi}{2} + \boldsymbol{i}_3 \sin\frac{\varphi}{2}\right).$$

The quantities characterizing the forms and dimensions of spacecraft orbits, initial and final orientations of spacecraft orbit are equal to ($\Omega_u^0 = \Omega_u(0)$, $I^0 = I(0)$, $\omega_\pi^0 = \omega_\pi(0)$; $\Omega_u^* = \Omega_u(t^*)$, $I^* = I(t^*)$, $\omega_\pi^* = \omega_\pi(t^*)$):

$$u_{\max} = 0.101907 \text{ m/sec}^2, \quad N^b = 0.35;$$

initial spacecraft position ($\varphi_0 = 3.940323$ rad):

$$\Omega_u^0 = 212.0^\circ, \quad I^0 = 63.0^\circ, \quad \omega_\pi^0 = 0.0^\circ;$$
$$\Lambda_0^0 = -0.235019, \quad \Lambda_1^0 = -0.144020,$$
$$\Lambda_2^0 = 0.502258, \quad \Lambda_3^0 = 0.819610;$$
$$\lambda_0^0 = -0.663730, \quad \lambda_1^0 = 0.518734,$$
$$\lambda_2^0 = -0.062608, \quad \lambda_3^0 = -0.535217;$$

final spacecraft position (it corresponds to the orientation of the orbit of GLONASS satellites):

$$\Omega_u^* = 215.25^\circ, \quad I^* = 64.8^\circ, \quad \omega_\pi^* = 0.0^\circ;$$
$$\Lambda_0^* = -0.255650, \quad \Lambda_1^* = -0.162241,$$
$$\Lambda_2^* = 0.510674, \quad \Lambda_3^* = 0.804694.$$

Scaling factors are equal to

$$R = 26000000 \text{ m}, \quad V = 2751.405874 \text{ m/sec},$$
$$T = 9449.714506 \text{ sec}.$$

The scaling factors correspond to spacecraft whose initial and final coordinates and velocity projections were taken from [25].

The parameters of the genetic algorithm are equal to

$$L = 40, \quad N_{\max} = 10000, \quad p_{mut} = 0.9.$$

In table 1 the results of the numerical solution of the problem for different numbers of the sections of the spacecraft active motion are presented.

TABLE 1. RESULTS OF THE GENETIC ALGORITHM

M	J	M	J
2	0.373953	7	0.496462
3	0.374595	8	0.513247
4	0.623807	9	0.696981
5	0.336295	10	1.257639
6	0.386022		

It was found that the functional J reaches its minimal value when there are five parts of spacecraft active motion.

Figures 1-4 present the results of numerical solution of the problem of reorientation of the spacecraft orbit.

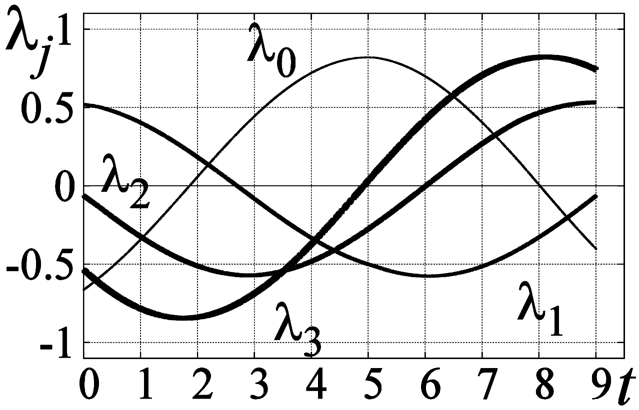

Fig. 1. Quaternion of orientation of the orbital coordinate system

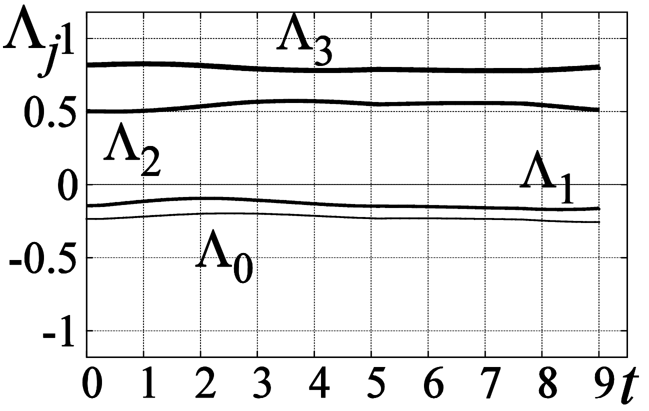

Fig. 2. Quaternion of orientation of the spacecraft orbit

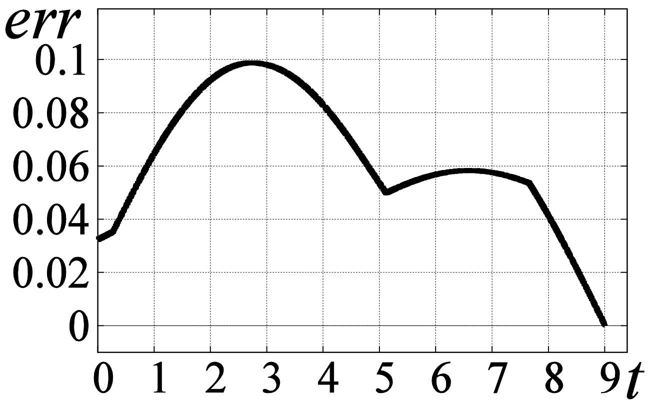

Fig. 3. Difference between current and desired spacecraft orbits

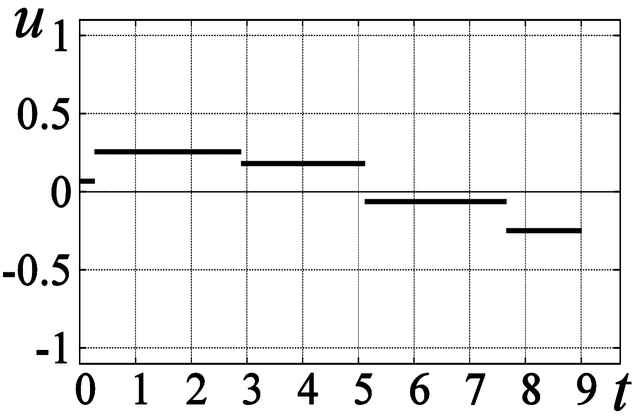

Fig. 4. Optimal control

In this case the time of flight of a controlled spacecraft is equal to 9.007084 dimensionless units. It corresponds to approximately 85114.37 sec (or 23.643 hours). We can see that the ranges of variation of the components of the quaternion of orientation of the orbit are smaller than the ranges for the components of the quaternion of orientation of the orbital coordinate system.

Note that duration of the first part of spacecraft active motion is smaller than other parts. But we should keep this part because solution with four parts of spacecraft active motion corresponds to bigger value of minimized functional.

V. Conclusion

In this paper we discussed the problem of spacecraft orbit reorientation. The proposed genetic algorithm can help us quickly find quasi-optimal solution of the problem. To obtain more optimal solution one can assume that the number of the parts of spacecraft active motion parts is not given. Next time we will try to modify the proposed algorithm to find this quantity.

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Ilia Alekseevich Pankratov,
Associate Professor of the Saratov State University,
Researcher of the Institute of Precision Mechanics and Control Problems of the Russian Academy of Sciences,
e-mail: PankratovIA@info.sgu.ru,
e-library.ru: authorid=608692.