Classification of natural numbers based on arithmetic progressions with a difference 6.

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Abstract—The article considers a classification of natural numbers based on the submission of the set of all natural numbers as union of six infinite arithmetic progressions. The classes themselves (bijective to the progressions) are considered as members of two finite semigroups with regard to the operations of addition and multiplication. The binary relations between classes and examples of natural numbers properties at such classification are given.

Keywords—Arithmetic progression, bijection, binary relations, classification, Goldbach prime, set of natural numbers, semigroup.

I. INTRODUCTION

The representation of the set of all natural numbers greater than or equal to 4 as the combination of six infinite arithmetic progressions denoted as $S_i = \{4 + 6m\}$, $S_5 = \{5 + 6m\} \cup S_6 = \{6 + 6m\}, S_7 = \{7 + 6m\} \cup S_8 = \{8 + 6m\}, S_9 = \{9 + 6m\}, m \in \mathbb{N}$ was considered in [1]-[3]. In this representation, every natural number $n$ has a unique expression in the form of pairs of numbers $(i_r, m)$, $i_r \in \mathbb{N}$, where $i_r$ is the index of progression, coinciding with the initial member of progression, and $m$ is the ordinal number of the number in this progression. It was also proved that all prime numbers $P$ greater than 4, $P > 4$ are contained only in two progressions of these six, namely $S_3 = \{5 + 6m\}$ and $S_7 = \{7 + 6m\}$.

Let us agree within the article not to introduce special notations for the proposed classes, which are bijective to six progressions, because substantial part lies in the number-the index of progression, which will be used in the expressions of binary relations between classes in the form of $6 \times 6$ tables.

In this article, when ensuring [1] approach, the following six progressions will be considered: $S_3 = \{3 + 6m\}; S_4 = \{4 + 6m\}; S_5 = \{5 + 6m\}; S_6 = \{6 + 6m\}; S_7 = \{7 + 6m\}; S_8 = \{8 + 6m\}$.

The initial fragment of this six progressions is given below (accordingly, and the offered classes natural) Fig.1. Primes in them are marked more in bold, and primes-twins also by underlining.

Properties of progressions go into properties of classes that act as elements of the semigroups under addition and multiplication. So, it should be noted the behavior of primes degrees within this six progressions included in the canonical multiplicative representation of numbers, equal to the index of progressions. So, for $S_3 = \{3, 9, 27, 81, 243, 729, 2187, \ldots\} \in S_3$, $S_4 = \{4, 16, 64, 256, 1024, 4096, \ldots\} \in S_4$, $S_5 = \{5, 125, 3125, 78125, \ldots\} \in S_5$, $S_6 = \{6, 36, 216, 1296, \ldots\} \in S_6$, $S_7 = \{7, 49, 343, 2401, \ldots\} \in S_7$, $S_8 = \{8, 32, 512, 2048, \ldots\} \in S_8$.

This property can be considered as an invariant of progression and, hence, an invariant of bijective class of the natural. On the other hand, it can be used as a marking of an infinite arithmetic progression by elements of an exponential nature (as automorphic built-in logarithmic scale).

1. $S_3$ contains all (even and odd) degrees of 3.
2. $S_4$ contains only even powers of 2.
3. $S_5$ contains only odd powers of 5.
4. $S_6$ contains all (even and odd) degrees of 6.
5. $S_7$ contains all (even and odd) degrees of 7.
6. $S_8$ contains only odd powers of 2.

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II. BINARY RELATIONS BETWEEN CLASSES AS ELEMENTS OF SEMIGROUPOS

Classes of natural, bijective to these progressions, are in binary relations to the operations of addition and multiplication, which are shown in a tabular form in Fig. 2. It means that for any pair of natural such that \( n_1 \in S_6 \) and \( n_2 \in S_6 \), the sum \( (n_1 + n_2) \in S_6 \), where \( r_3 \) is the index at the intersection of the \( r_3 \)-th column and the \( r_3 \)-th row (Fig. 2a) (or vice versa—the row and the column, since both addition and multiplication are commutative), and the tables are symmetric. Similarly, for multiplication (Fig. 2b).

In the semigroup under addition the role of the "0" makes the class \( S_6 \), and in the semigroup under multiplication the role of "1" performs the class \( S_6 \).

Given in the form of tables binary relations between classes can be used to calculate tables for higher arities. Here we will limit ourselves to the example tables of ternary relations between classes. Such tables are no longer flat but three-dimensional \( 6 \times 6 \times 6 \) (Fig. 3). For a flat rendering will use the so-called "slice-type".

We are interested in the question: To which class belongs the operation result of "random" pairs or triples of natural. It was natural therefore to consider the table of binary relations as the set of possible outcomes at a probability of 1/6 of each class of the operand. Then the ratio of the number of indexes of each class to 36 is equal to the probability to get the result in this class of natural.

For binary relations under addition the probability of the result from each class is equal to 1/6, since \(#(r, T_6) = 6\). For binary relations under multiplication we have different picture, because:

\[
#(3, T_6) = 5; \quad #(4, T_6) = #(8, T_6) = 6; \quad #(5, T_6) = #(7, T_6) = 2; \\
#(6, T_6) = 15
\]

Hence the probability of the product of two randomly selected natural to be in the class \( p_i (r, b) \):

\[
p_i(3, b) = 5 / 36; \quad p_i(4, b) = p_i(8, b) = 1 / 6; \\
p_i(5, b) = p_i(7, b) = 1 / 18; \quad p_i(6, b) = 5 / 12;
\]

Similar calculations for ternary relations under multiplication give the following results:

\[
#(3, T_6) = 19; \quad #(4, T_6) = #(8, T_6) = 28; \quad #(5, T_6) = #(7, T_6) = 4; \\
#(6, T_6) = 133
\]

Probability to the product of three randomly selected natural to be in the class \( p_i (r, t) \):

\[
p_i(3, t) = 19 / 216; \quad p_i(4, t) = 7 / 54; \quad p_i(5, t) = 1 / 54; \\
p_i(6, t) = 133 / 216; \quad p_i(7, t) = 1 / 54; \quad p_i(8, t) = 7 / 54;
\]

At the construction of natural of some random class in the degree the response to a similar question about the probability of the class for the result is contained in the table on the main diagonal. In particular for the construction of the natural square (a binary relation under multiplication) the diagonal is a set \( D_{2b} = \{S_1, S_2, S_3, S_3, S_3, S_3, S_3, S_3\} \). \( S_3, S_3 \notin D_{2b} \), which corresponds to (1).

Thus \( p_i(3, b) = 1 / 6; \quad p_i(4, b) = 1 / 3; \quad p_i(5, b) = 0; \\
p_i(6, b) = 1 / 6; \quad p_i(7, b) = 1 / 3; \quad p_i(8, b) = 0; \)

Similarly:

\[
D_3 = \{S_3, S_4, S_7, S_7, S_7, S_7, S_7, S_7\} \quad \text{and} \quad p_i(3, t) = p_i(4, t) = ... = p_i(8, t) = 1 / 6;
\]
III. NEW CLASSIFICATION AND THE GOLDBACH PRIMES

Prime pairs $g_1$ and $g_2$, the total of which is equal to a given even natural number, play a significant role in researches related to the binary Goldbach problem. These pairs are called the Goldbach primes.

In the proposed classification, as shown in [1], all primes greater than 4 belong to the classes $S_5$ and $S_7$. As well as compound of these classes they satisfy to the binary relations under addition (Fig.2). All even greater than or equal to 4 belong only to the classes $S_4$, $S_6$, $S_8$. It is easy to see (Fig.4) that even from the class $S_4$ are Goldbach prime numbers conjecture only from $S_5$ ($G(S_5) \in S_4$), and the even-numbered from $S_8$ only from $S_7$ ($G(S_7) \in S_8$). Even from $S_6$ always have one Goldbach prime from $S_5$, and the other from $S_7$, including this applies to primes-twins.

IV. CONCLUSION

The General structure of the proposed classification is shown in Fig.5, in which:

$P(S_5)$ — primes of the class $S_5$; $P(S_7)$ — primes of the class $S_7$; $P(S_4 \cup S_5)$ — twin primes;

REFERENCS

