On composition of infinitary structures and symmetries between primes.

G.G. Ryabov, V.A. Serov

***Abstract*—In the article on the level of presentation, close to elementary deals with the composition structures of an n-cube, a global k-ary trees and natural numbers N. The properties of this composition are used when considering the symmetry of primes in the structure of natural N on the basis of differential tabloid defined on $P \times N$, where P is set of odd primes.**

***Keywords*—finite alphabet, n-cube, bijection, k-faces, the ternary matrix, k-ary tree, the recursion, k-tuples of natural numbers, primes.**

I. Introduction

Intensive research in recent time [1-3], the fundamental problems of number theory not in the least connected with hope to create effective tools not only for this area, but also to solve the problems of computability in General, and thereby it is possible to look to the future structures of computers. No coincidence with these studies involve the whole international groups (Polymath, Magma, Pari ets.) that develops software in these and related fields. In this brief article an attempt is made to find the contours of the development of indirection, which is closely related to various patterns. We are talking about the structure, combining the shortest paths in n-cube, k-ary global trees and set of natural N. This is all in line with the concept of Y. I. Manin on the creation of constructive universes [4].

II. The steps of designing the composition

There is graphical pattern of such composition below (Fig.1).

1. In [5-6] were proposed, the bijection k-faces of an n-cube as the set of words A_n^* over the alphabet $A = \{0,1,2\}$, and algebra over words A_n^*, metric Hausdorff-Hamming, calculated in words.
2. A bijection k shortest paths between antipodal vertices of an n-cube, and ternary symbolic $n \times (n-k+1)$ matrices with row-words from A_n^* (hereafter TSM) was established [7-8].
3. The set TSMD diagonal matrices of the form-bijections k shortest paths between the vertices (00...0) and (11...1) n-cube was introduced. TSMD is defined on the base of dimension k and the decomposition recursion [9].
4. TSMD were mapped at the vertices of the global k-ary tree GKT (the root degree is k, the remaining vertices realize relations of the "parent- k children")—genetic component of the composition. In accordance with automorphic function TSMD, for each matrix is uniquely calculated its number in the sequence of natural N and thus are numbered the vertices of the GKT (Fig.1).
5. The representation of a sequence as a natural chain of k-tuples complements the overall structure TSMD-GKT-N for a given k. So the chains of k-tuples for $k = 3,5,7$ (first odd primes) are next:

$T(3) = \{<1,2,3>, <4,5,6>, <7,8,9>, \ldots\}; (Fig.1)$

$T(5) = \{<1,2,3,4,5>, <6,7,8,9,10>,$
$<11,12,13,14,15>, \ldots\};$

$T(5) = \{<1,2,3,4,5,6,7>, <8,9,10,11,12,13,14>,$
$<15,16,17,18,19,20,21>, \ldots\};$

$T(11); T(13); T(17); \ldots$

Every natural for each T ($k = p_i$) where $p_i \in P$ —set of odd primes, has a unique number to the tuple ($1,2,3,\ldots,N$) and unequivocal position inside the k-tuple ($1,2,3,\ldots,k$). Each level (generation) g of the k-ary tree contains $k^{(g-1)}$ tuples (the root has level g=0 and by definition for him $k^{-1} = 0$). Thus is implemented a bunch of "mathematical logic—representation theory—metric geometry—combinatorics—number theory" within a single composite structure.

III. About symmetry of primes

We will show that this gives a compositional structure when considering properties of symmetry of primes in the structure of natural N.

As an indicator of the mutual arrangement of primes and natural table introduces differential DT ($P \times N$), each cell (p_i, n_s) which is designed to store the difference of the $d_{si} = (n_s - p_i)$, $d_{si} \in Z$. So $d_{si} \geq 0$ for $n_s \geq p_i$ (left primes for n_s) and $d_{si} < 0$ при $n_s < p_i$ (right primes for n_s).

If $|d_{si}| = |d_{sj}|$, then this means that a pair primes equidistant for n_s (p_i and p_j) are symmetric relative to n_s. If for d_{si} (left prime p_i) equidistant of the pair corresponds to the right not prime p_j (i.e., the compound corresponds to the right number), then the left prime p_i does not have a symmetric right. The lack of symmetrical

Manuscript received December 1, 2015.

G.G. Ryabov is with the Research Computing Center, M.V. Lomonosov Moscow State University, Moscow, Russia (e-mail: gen-ryabov@yandex.ru).

V.A. Serov is with the Research Computing Center, M.V. Lomonosov Moscow State University, Moscow, Russia.

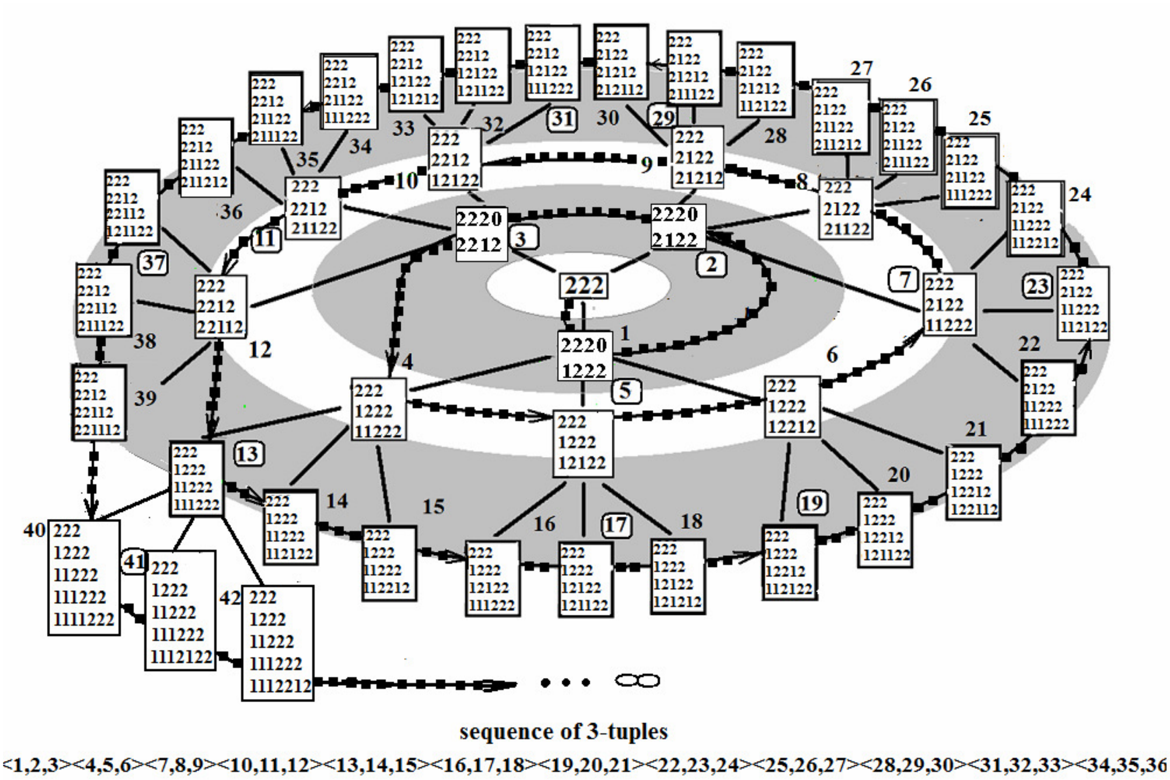

Fig.1. Composite structure STMD-GKT- N for k=3 and levels g=0 (the 3-path in 3-cube is 3-cube), g=1 (3-paths in 4-cube), g=2 (3-paths in 5-cube) etc. Zero elements in the matrices are omitted. Natural numbers next to the matrices and vertices GKT is the result of automorphic function for each STMD. Simple framed. Edges in GKT—straight line segments, an endless spiral of natural marked by a thick dotted line.

right and can be installed without calculations, when $d_{sj} < 0$. The analysis of mutual positions in k-tuples "of the left prime p_i (denoted by x_1), for natural n_s (denoted by x_2) and a possible candidate for what's right $p_j(x_3)$ leads to the following proposition.

x_2 may correspond to the position $|x_2 - x_1| = |x_2 - x_3|$ then, when the positions x_1 and x_2 are not solutions of the linear congruence modulo k:

$$2x_2 - x_1 \equiv 0 \,(mod\, k), \qquad (1)$$

for all $k = p_i, p_i \le 2n_s - 3.$

This statement can be regarded as a criterion incompatibility of positions in the tuples for pairs of symmetric primes with respect to this natural. The set of solutions congruence (1) will be denoted by $H(k = p_i)$ and present some partial solutions (for $p_i = 3,5,7,11$) and its General appearance.

$H(3) = \{(1,2),(2,1),(3,3)\};$

$H(5) = \{(1,3),(2,1),(3,4),(4,2),(5,5)\};$

$H(7) = \{(1,4),(2,1),(3,5),(4,2),(5,6),(6,3),(7,7)\};$

$H(11) = \{(1,6),(2,1),(3,7),(4,2),(5,8),(6,3),(7,9),$
$(8,4),(9,10),(10,5),(11,11)\};$

P \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
3				1	2		4	5		7	8		10			13	14			17		19	20		22	
5						1		3	4		6	7		9	10		12	13			16		18	19		21
7					-2	-1			2	3		5	6		8	9		11	12			15		17	18	
11							-4	-3	-2			1		3	4		6	7		9	10			13		15
13								-5	-4	-3		-1			2	3		5			8	9			12	
17										-7	-6	-5		-3	-2			1		3	4		6	7		
19											-8	-7	-6		-4	-3		-1			2			5	6	
23													-10	-9	-8		-6	-5		-3	-2					3
29																-13	-12	-11		-9	-8		-6	-5		-3
31																	-14	-13	-12		-10	-9		-7	-6	
37																				-17	-16	-15		-13	-12	
41																						-19	-18	-17		-15
43																							-20	-19	-18	
47																									-22	-21
53																										
59																										
61																										
67																										
71																										
73																										
79																										
83																										
89																										
97																										

P \ N	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
3		25	26		28	29			32		34	35			38		40			43				47	
5			24			27	28			31		33	34			37		39			42				46
7	20			23			26	27			30		32	33			36		38			41			
11	16		18			21			24	25			28		30	31			34		36			39	
13	14	15		17			20	21		23	24			27		29	30			33		35			38
17	10		12	13		15			18			21	22			25		27	28			31		33	
19		9		11	12		14			17			20	21			24		26	27			30		32
23	4			7		9	10		12			15			18	19			22		24	25			28
29				1			4		6	7		9			12			15	16			19		21	22
31	-4			-1				3		5	6		8			11			14	15			18		20
37	-10	-9		-7			-4	-3					2	3		5			8			11	12		
41	-14		-12	-11		-9			-6	-5			-2			1		3			6			9	10
43	-16	-15		-13	-12		-10			-7	-6			-3		-1			2			5			8
47	-20		-18	-17		-15	-14		-12			-9	-8			-5		-3	-2					3	
53		-25	-24	-23		-21	-20		-18	-17		-15			-12	-11			-8		-6	-5		-3	
59					-28	-27	-26		-24	-23		-21	-20		-18			-15	-14			-11		-9	-8
61						-29	-28	-27		-25	-24		-22	-21		-19			-16	-15			-12		-10
67									-32	-31	-30		-28	-27		-25	-24		-22			-19	-18		
71											-34	-33	-32		-30	-29		-27	-26		-24			-21	-20
73												-35	-34	-33		-31	-30		-28	-27		-25			-22
79															-38	-37	-36		-34	-33		-31	-30		-28
83																	-40	-39	-38		-36	-35		-33	-32
89																				-43	-42	-41		-39	-38
97																								-47	-46

Fig.2. Fragment of DT. The dark cells correspond the decisions of congruence (1).

Let be $m = (p_i + 1)/2$, then:

$$H(p_i) = \{(1,m),(2,1),(3,m+1),(4,2),..., (p_i - 2, p_i - 1),(p_i - 1, m - 1),(p_i, p_i)\}; \quad (2)$$

$$|H(p_i)| = p_i;$$

Consistently using this criterion in DT you can shade the whole bunch of cells do not have pairs of primes, as already done in Fig.2. This forecast "antisymmetrical" is quite deterministic.

IV. A BRIEF REMARK ABOUT THE RANDOMNESS OF PRIMES

One of the widely used methods in the study of behavior of primes are methods involving a random process or processes underlying the patterns of primes numbers among natural. So in June 2015 at the international seminar "Globus" report of Professor Kevin Ford was called "the primes play dice?"

Returning to the result of the previous section, consider the mapping solutions $H(p_i)$ comparing (1) for each p_i on $p_i \times p_i$ square board.

Each pair of incompatible positions (x_1, x_2) corresponds to a cell on this board, we will consider it black. Then the overall picture of solutions of (1) will have the form presented in Fig.3.

This mapping encourages us to answer: «If primes and play, not dice, and chess on the $p_i \times p_i$ boards and is one of the most exotic figures—horse».

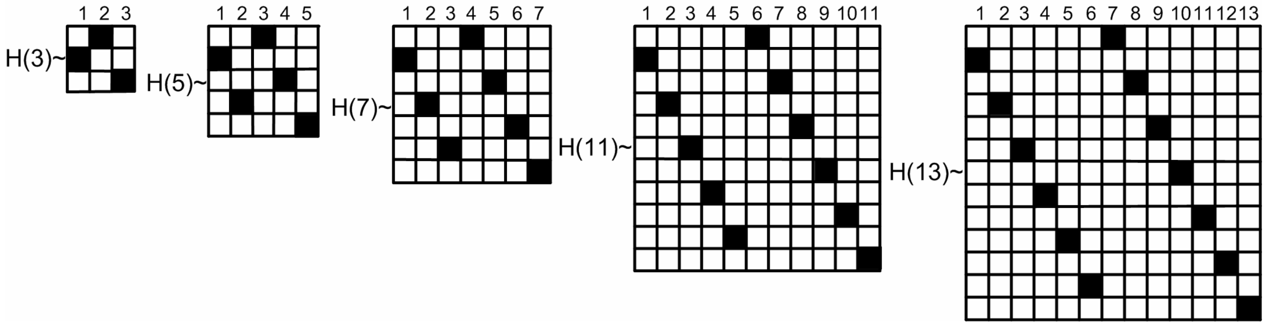

Fig.3. Quasi check-mate boards and mapping of decisions (2) on them.

V. CONCLUSION

Composition infinitary or similar structures (by similarity briefly discussed above) is likely to play a key role in addressing the biological, economic and social problems. Especially in the study of the ergodic behaviour of one structure inside another (co-structural behavior), until the minimal representations at the level of Young diagrams and tables. So the question is how to respond to this architecture of future computer systems using coprocessors to work effectively with a wide range of bijections, where the role of exact arithmetic for huge integers in different number systems, may largely be the determining factor.

REFERENCES

[1] K. Ford, B. Green, S. Konyagin, J. Maynard, T. Tao, “Long gaps between primes,” 2015. Available: http://arxiv.org/pdf/1412.5029v2

[2] D.H.J. Polymath, “The "bounded gaps between primes" Polymath project - a retrospective,” Sep 2014. Available: http://arxiv.org/pdf/1409.8361v1

[3] Janos Pintz, “Patterns of primes in arithmetic progressions,” Sep 2015. Available: http://arxiv.org/pdf/1509.01564v2

[4] Yuri I.Manin, “Classical computing, quantum computing and Shor`s factoring algorithm,” 1999. Available: http://arxiv.org/pdf/quant-ph/9903008.pdf

[5] G.G. Ryabov, “On quaternary coding of cubic structures,” Numerical methods and programming, vol. 10, no. 2, pp. 340-347, 2009. Available (in russian): http://num-meth.srcc.msu.ru/zhurnal/tom_2009/pdf/v10r138.pdf

[6] G.G. Ryabov, “Hausdorff metrics on faces of n-cube,” Fundamental and applied mathematics, vol. 16, no. 1, pp. 151-155, 2010. DOI: 10.1007/s10958-011-0487-3. Available (in russian): http://mech.math.msu.su/~fpm/ps/k10/k101/k10112.pdf

[7] G.G. Ryabov, V.A. Serov, “On classification of k-dimension paths in n-cube,“ Applied Mathematics (SCIRP), vol. 5, no. 4, pp. 723-727, 2014. DOI: 10.4236/am.2014.54069. Available: http://dx.doi.org/10.4236/am.2014.54069

[8] G.G. Ryabov, V.A. Serov, “ “Multidimensional metro” and symbol matrices,” International Journal of Open Information Technologies, vol. 2, no. 11, pp. 10-18, 2014. Available (in russian): http://injoit.org/index.php/j1/article/view/157/116

[9] G.G. Ryabov, V.A. Serov, “Polymorphism of ternary symbolic matrixes and genetic space of shortest k-paths in n-cube,” International Journal of Open Information Technologies, vol. 3, no. 7, pp. 1-11, 2015. Available (in russian): http://injoit.org/index.php/j1/article/view/214/173