Robust Adaptive Fixed-Time Terminal Sliding Mode Control with Disturbance Observer for Nonlinear Systems

Hoang Duc Long

***Abstract*-This paper proposes a robust and efficient control strategy for a class of second-order uncertain nonlinear systems by integrating Adaptive Fixed-Time Terminal Sliding Mode Control (AFTSMC) with a low-pass filtered Disturbance Observer (DO), forming the novel AFTSMC-DO framework. The proposed approach addresses key challenges in nonlinear system control: fast convergence, chattering mitigation, and disturbance rejection. The AFTSMC design leverages a non-singular fast terminal sliding manifold, ensuring that system trajectories converge to the desired reference in a fixed time independent of initial conditions. To further improve disturbance rejection and reduce reliance on high control gains, a DO is introduced to estimate unknown matched disturbances using filtered state derivatives, enhancing both transient performance and robustness. A rigorous Lyapunov-based analysis proves the global fixed-time stability of both the tracking errors and disturbance estimation error. The disturbance bound is adaptively estimated using a barrier Lyapunov function (BLF), ensuring bounded and smooth adaptation. Theoretical guarantees are supported by a comprehensive comparative simulation on the inverted pendulum on a cart-a classic benchmark of underactuated nonlinear dynamics. The proposed AFTSMC-DO is compared with the Adaptive Full-Order Time-Varying Sliding Mode Control (AFOTVSMC). Simulation results show that AFTSMC-DO achieves faster convergence, lower steady-state error, better tracking under time-varying disturbances, and lower control effort. These advantages make AFTSMC-DO a promising candidate for applications in robotics, aerospace, and industrial systems requiring rapid and robust control under uncertainty.**

***Keywords*-Adaptive Control, Fixed-Time Stability, Terminal Sliding Mode, Disturbance Observer, Nonlinear Systems, Robust Tracking, Inverted Pendulum on Cart.**

I. INTRODUCTION

Sliding Mode Control (SMC) has been widely recognized for its robustness in handling nonlinear systems subject to matched uncertainties and disturbances [1]. Despite its benefits, traditional SMC suffers from the reaching phase and high-frequency chattering [2], which degrades performance in practical systems [3, 4]. To overcome this, Terminal Sliding Mode Control (TSMC) introduces nonlinear manifolds to guarantee finite-time convergence, improving transient dynamics [5, 6].

Fixed-Time Terminal Sliding Mode Control (FTSMC) extends this idea by ensuring that convergence time is uniformly bounded, independent of initial conditions-a desirable property for time-critical applications [7, 8]. Recent studies [9-12] have explored adaptive FTSMC strategies to deal with time-varying or unknown disturbance bounds. However, most existing many adaptive laws rely on overestimating the uncertainty bound or converge slowly in the presence of rapidly changing uncertainties [13, 14].

On the other hand, Disturbance Observers (DOs) have been proposed to estimate and compensate for unknown perturbations, improving robustness without excessive control effort [15, 16]. In particular, filtering-based DOs provide practical simplicity compared to high-order observers [17, 18]. Integrating DOs into sliding mode frameworks has shown promise in rejecting matched disturbances and improving control smoothness [19-21].

Nevertheless, there remains a gap in unifying the benefits of adaptive FTSMC with disturbance observers into a single provably stable framework. Motivated by this, we propose a novel Adaptive Fixed-Time Terminal Sliding Mode Control with Disturbance Observer (AFTSMC-DO). Unlike prior work, AFTSMC-DO:

- Ensures fixed-time convergence regardless of initial conditions;
- Estimates matched disturbances using a low-pass filtered DO;
- Adapts the uncertainty bound using a smooth Barrier Lyapunov Function (BLF).

The contributions of this paper are threefold:

1. A unified AFTSMC-DO control law that combines fixed-time convergence, disturbance estimation, and smooth adaptation.
2. A Lyapunov-based fixed-time stability proof incorporating the DO error dynamics.
3. Comparative validation of AFTSMC-DO, AFOTVSMC on an inverted pendulum benchmark.

The remainder of this paper is structured as follows: Section 2 reviews the Adaptive Full-Order Time-Varying Sliding Mode Control (AFOTVSMC) approach and its stability analysis. Section 3 introduces the proposed Adaptive Fixed-Time Terminal Sliding Mode Control with Disturbance Observer (AFTSMC-DO), including the design of the disturbance observer and a rigorous stability proof. In Section 4, the effectiveness of AFTSMC-DO is demonstrated through simulation on an inverted pendulum on a cart, with performance compared against AFOTVSMC. Finally, Section 5 concludes the paper with key findings and suggestions for future research.

Manuscript received 09.07.2025.

Hoang Duc Long, Lecturer at Faculty of Control Engineering, Le Quy Don Technical University, Hanoi, Vietnam (corresponding author, phone: (+84)915961451; e-mail: longhd@lqdtu.edu.vn).

II. ADAPTIVE FULL-ORDER TIME-VARYING SLIDING MODE CONTROL

Consider a class of second-order uncertain nonlinear systems: [22]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x,t) + b(x,t)u(t) + \Delta(x,t) \end{cases} \tag{1}$$

where $x_1(t) = \left[x_{11}(t), \ldots, x_{1n}(t)\right]^T \in \mathbb{R}^n$ and $x_2(t) = \left[x_{21}(t), \ldots, x_{2n}(t)\right]^T \in \mathbb{R}^n$ are vectors of state variables; $x(t) = [x_1, x_2]^T \in \mathbb{R}^{2n}$; $f(x,t) \in \mathbb{R}^n$ is smooth known dynamics; $b(x,t) \in \mathbb{R}^{n\times n}$ is invertible and bounded: $0 < b_{\min} \le |b(x,t)| \le b_{\max}$; $u(t) \in \mathbb{R}^n$ is the control input; $\Delta(x,t) \in \mathbb{R}^n$ is bounded lumped uncertainties $\left(|\Delta(x,t)| \le \Delta_{\max}\right)$ that includes model uncertainties $\Delta f(x,t) \in \mathbb{R}^n$ and external bounded disturbances $d(x,t) \in \mathbb{R}^n$.

Assumption 1. The lumped uncertainty $\Delta(x,t)$ and its time derivative are bounded: $\|\Delta(x,t)\| < \mu$ and $\|\dot{\Delta}(x,t)\| < \rho$.

Assumption 2. The reference signal $x_d(t) \in \mathbb{R}^n$ is smooth, and its derivatives $\dot{x}_d(t) \in \mathbb{R}^n$ and $\ddot{x}_d(t) \in \mathbb{R}^n$, are bounded and exits for all $t \in \mathbb{R}_+$.

The tracking errors are defined:

$$\begin{cases} e_1(t) = x_1(t) - x_d(t) \\ e_2(t) = x_2(t) - \dot{x}_d(t) \end{cases} \tag{2}$$

The time-varying sliding surface is defined:

$$s(t) = e_2(t) + \dot{\lambda}(t) + k\left(e_1(t) + \lambda(t)\right) \tag{3}$$

where $s(t) = \left[s_1(t), \ldots, s_n(t)\right]^T \in \mathbb{R}^n$ is the vector of sliding surfaces; $k = \mathrm{diag}(k_1, \ldots, k_n) \in \mathbb{R}^{n\times n}$ is a positive-definite matrix; $\lambda(t) = \left[\lambda_1(t), \ldots, \lambda_n(t)\right]^T \in \mathbb{R}^n$ is a time-varying piecewise function that is given as below:

$$\lambda_i(t) = \begin{cases} \lambda_{ig}(t) & t \le t_f \\ 0 & t > t_f \end{cases}, \quad i = \overline{1,n} \tag{4}$$

The fast terminal sliding manifold is designed:

$$\begin{aligned} \sigma(t) = \dot{s}(t) + c_1\vartheta(s(t))\arctan(s(t)) \\ + c_2\vartheta(s(t))\arctan^{\eta}(s(t)) \end{aligned} \tag{5}$$

where $c_j = diag\left(c_{j1}, \ldots, c_{jn}\right) \in \mathbb{R}^{n\times n}$, $(j = 1,2)$ are positive definite matrices; $\vartheta(s) = diag\left(1+s_1^2, \ldots, 1+s_n^2\right) \in \mathbb{R}^{n\times n}$; $\arctan(s) = \left[\arctan(s_1), \ldots, \arctan(s_n)\right]^T \in \mathbb{R}^n$; $0 < \eta < 1$, $\arctan^{\eta}(s) = \left[\arctan^{\eta}(s_1), \ldots, \arctan^{\eta}(s_n)\right]^T \in \mathbb{R}^n$.

The control input is proposed:

$$u(t) = -b^{-1}(x,t)\left(u_{eq}(t) + u_{sw}(t)\right) \tag{6}$$

where the equivalent control $u_{eq}(t)$ is designed as:

$$\begin{aligned} u_{eq}(t) = f(x,t) + \ddot{\lambda}(t) + k\left(e_2(t) + \dot{\lambda}(t)\right) - \ddot{x}_d(t) \\ + c_1\vartheta(s)\arctan(s(t)) + c_2\vartheta(s(t))\arctan^{\eta}(s(t)) \end{aligned} \tag{7}$$

and the switching control $u_{sw}(t)$ is designed as:

$$\dot{u}_{sw}(t) = \left(a + \hat{\rho}(t)\right)\mathrm{sign}\left(\sigma(t)\right) \tag{8}$$

where a is a small constant; $\hat{\rho}(t)$ is the estimate of ρ that is designed as below:

$$\dot{\hat{\rho}}(t) = \begin{cases} 0, & \|\sigma(t)\| \le \upsilon \\ \dfrac{1}{\chi}\|\sigma(t)\|, & \text{otherwise} \end{cases} \tag{9}$$

where $\chi > 0$.

Stability of AFOTVSMC:

The Lyapunov candidate:

$$V_1 = \frac{1}{2}\sigma^2 + \frac{1}{2}(\Delta - \hat{\rho})^2 \tag{10}$$

The derivative of V_1:

$$\dot{V}_1 = \sigma\dot{\sigma} + (\Delta - \hat{\rho})\dot{\hat{\rho}} \tag{11}$$

From (3), (5), (6):

$$\dot{\sigma} = -\hat{\rho}\mathrm{sign}(\sigma) + \Delta \tag{12}$$

So that:

$$\begin{aligned} \dot{V}_1 &= \sigma\left(-\hat{\rho}\mathrm{sign}(\sigma) + \Delta\right) + (\Delta - \hat{\rho})\dot{\hat{\rho}} \\ &\le -\hat{\rho}|\sigma| + |\Delta||\sigma| + (\Delta - \hat{\rho})\dot{\hat{\rho}} \end{aligned} \tag{13}$$

Case 1: $|\sigma| \le \upsilon$

$$\dot{V}_1 \le -\left(\hat{\rho} - |\Delta|\right)|\sigma| < 0, \quad \text{if } \hat{\rho} > \Delta_{\max} \tag{14}$$

Case 2: $|\sigma| > \upsilon$

$$\begin{aligned} \dot{V}_1 &\le -\hat{\rho}|\sigma| + |\Delta||\sigma| + (\Delta - \hat{\rho})\frac{1}{\chi}|\sigma| \\ &\le -\left(1 + \frac{1}{\chi}\right)|\sigma|\left(\hat{\rho} - |\Delta|\right) < 0, \quad \text{if } \hat{\rho} > \Delta_{\max} \end{aligned} \tag{15}$$

Therefore, $\dot{V}_1 < 0$. Hence, the sliding variable $\sigma(t)$ converges to 0 in finite time, tracking errors $e_1(t)$ and $e_2(t)$ converge to zero in finite time.

III. ADAPTIVE FIXED-TIME TERMINAL SLIDING MODE CONTROL BASED ON DISTURBANCE OBSERVER

A. *Disturbance Observer*

Use an observer with low-pass filtering:

$$\hat{\Delta}(x,t) = \frac{1}{\tau p + 1}\left(\dot{x}_2(t) - f(x,t) - b(x,t)u(t)\right) \tag{16}$$

where p is the Laplace operator.

B. *Adaptive Fixed-Time Terminal Sliding Mode Control*

A fixed-time sliding surface is defined:

$$s(t) = e_2(t) + \alpha_1 e_1(t) + \alpha_2 e_1^{\frac{p}{q}}(t) \tag{17}$$

where p and q are positive odd integers and $p > q > 1$.

The derivative of $s(t)$:

$$\dot{s}(t)=\dot{x}_2(t)-\ddot{x}_d(t)+\alpha_1 e_2(t) +\alpha_2\frac{p}{q}|e_1(t)|^{\frac{p-q}{q}}e_2(t)\operatorname{sign}(e_1(t)) \tag{18}$$

The non-singular terminal sliding manifold:

$$\sigma(t)=\dot{s}(t)+k_1|s(t)|^{\gamma_1}\operatorname{sign}(s(t)) +k_2|s(t)|^{\gamma_2}\operatorname{sign}(s(t)) \tag{19}$$

where $0<\gamma_1<1<\gamma_2$; $k_1,k_2>0$.

The derivative of $\sigma(t)$:

$$\dot{\sigma}=\ddot{s}+k_1\gamma_1|s|^{\gamma_1-1}\dot{s}+k_2\gamma_2|s|^{\gamma_2-1}\dot{s} \tag{20}$$

The control law is designed as:

$$u(t)=-b^{-1}(x,t)\begin{pmatrix} f(x,t)+u_s(t)+ \\ \hat{\rho}(t)\operatorname{sat}\left(\dfrac{\sigma(t)}{\delta}\right)+\hat{\Delta}(x,t)\end{pmatrix} \tag{21}$$

where $\hat{\rho}(t)$ is adaptive estimate of the uncertainty bound that is defined later; δ is a small constant for the saturation function $\operatorname{sat}(\cdot)$; $u_s(t)$ is

$$u_s(t)=\ddot{x}_d(t)-\alpha_1 e_2(t)-\alpha_2\frac{p}{q}e_1^{\frac{p-q}{q}}(t)e_2(t) +k_1|s(t)|^{\gamma_1}\operatorname{sign}(s(t))+k_2|s(t)|^{\gamma_2}\operatorname{sign}(s(t)) \tag{22}$$

Barrier Lyapunov Function:

$$W(t)=-\ln\left(1-\frac{\hat{\rho}(t)}{\rho_{\max}}\right),\quad \text{with } \hat{\rho}(t)<\rho_{\max} \tag{23}$$

The derivative of $W(t)$:

$$\dot{W}(t)=\frac{\dot{\hat{\rho}}(t)}{\rho_{\max}-\hat{\rho}(t)} \tag{24}$$

The adaptive law based on the Barrier Lyapunov Function (BLF) is defined as::

$$\dot{\hat{\rho}}(t)=\frac{1}{\chi}\left(\rho_{\max}-\hat{\rho}(t)\right)|\sigma(t)| \tag{25}$$

where $\rho_{\max}$ is a conservative but finite upper bound for $\hat{\rho}(t)$; $\chi>0$ is a tuning constant. This adaptive law ensures smooth and bounded adaptation of the disturbance estimate and fits naturally into Lyapunov-based fixed-time stability analysis.

Theorem 1. *Consider the system under Assumptions 1-2. If the control law and adaptation law are applied as designed, then the tracking errors $e_1(t)\to 0, e_2(t)\to 0$ and the slide variable $s(t)\to 0,\sigma(t)\to 0$ in fixed time, independent of initial conditions.*

Proof.

The Lyapunov candidate is chosen as:

$$V(t)=\frac{1}{2}\sigma^2(t)+W(t) \tag{26}$$

where $W(t)$ is Barrier Lyapunov Function (23).

Differentiate $V(t)$:

$$\dot{V}(t)=\sigma(t)\dot{\sigma}(t)+\frac{\dot{\hat{\rho}}(t)}{\rho_{\max}-\hat{\rho}(t)} \tag{27}$$

From (18), the derivative of $\ddot{s}(t)$:

$$\ddot{s}(t)=\ddot{x}_2(t)-\dddot{x}_d(t)+\alpha_1\left(\dot{x}_2(t)-\ddot{x}_d(t)\right) +\alpha_2\frac{p}{q}\begin{pmatrix}\dfrac{p-q}{q}|e_1(t)|^{\frac{p-2q}{q}}e_2^2(t)\left(\operatorname{sign}(e_1(t))\right)^2 \\ +|e_1(t)|^{\frac{p-q}{q}}\left(\dot{x}_2(t)-\ddot{x}_d(t)\right)\operatorname{sign}(e_1(t))\end{pmatrix} \tag{28}$$

Combine (1), (21), (28) and (20), we have

$$\dot{\sigma}(t)=-\hat{\rho}(t)\operatorname{sign}(\sigma(t))+\tilde{\Delta}(x,t) \tag{29}$$

where $\tilde{\Delta}(x,t)=\Delta(x,t)-\hat{\Delta}(x,t)$.

Substitute (29) into (27):

$$\begin{aligned}\dot{V}(t)&=\sigma(t)\tilde{\Delta}(x,t)-\hat{\rho}(t)|\sigma(t)|+\frac{1}{\chi}|\sigma(t)| \\ &\le|\sigma(t)|\left(\left|\tilde{\Delta}(x,t)\right|-\hat{\rho}(t)+\frac{1}{\chi}\right)\end{aligned} \tag{30}$$

Choose χ such that:

$$\hat{\rho}(t)>\max\left|\tilde{\Delta}(x,t)\right|+\frac{1}{\chi} \tag{31}$$

Therefore, $\dot{V}(t)<0$. Hence, the system is globally uniformly stable, $\sigma(t)\to 0$ and $s(t)\to 0$, $e_1(t)\to 0$ and $e_2(t)\to 0$.

■

IV. SIMULATION RESULTS

In this section, the proposed controller will be applied for an inverted pendulum on a cart that is illustrated in Fig. 1.

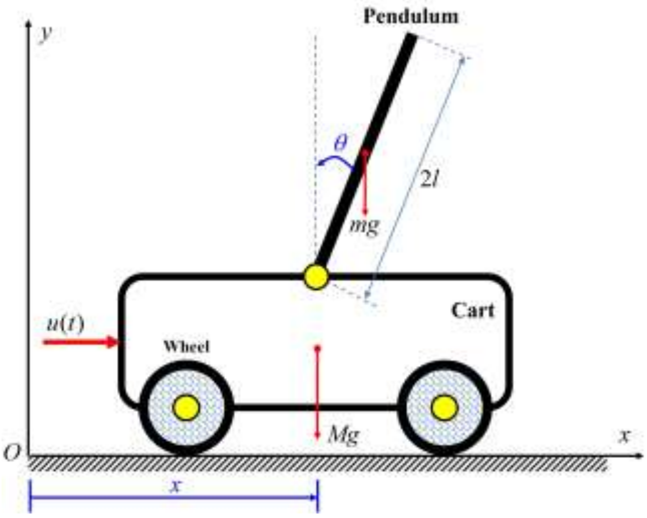

Fig. 1. Model of Inverted Pendulum on Cart.

The mathematic model of the inverted pendulum model is described in the state-space model as below [23]

$$\begin{cases}\dot{x}_1=x_2 \\ \dot{x}_2=f(x,t)+b(x,t)u+\Delta(x,t)\end{cases} \tag{32}$$

where θ is the pendulum angle from vertical (down) (rad); x is cart position coordinate (m); m_c is mass of the cart

(kg); m_p is mass of the pendulum (kg); $2l$ is length of the pendulum (m); g is the gravity acceleration $(kg.m^2)$; u is the control law; $f(x,t)$ and $b(x,t)$ are defined:

$$f = \frac{g\sin x_1 - \dfrac{m_p l x_2^2 \cos x_1 \sin x_1}{m_c + m_p}}{l\left(\dfrac{4}{3} - \dfrac{m_p \cos^2 x_1}{m_c + m_p}\right)};\ b = \frac{\dfrac{\cos x_1}{m_c + m_p}}{l\left(\dfrac{4}{3} - \dfrac{m_p \cos^2 x_1}{m_c + m_p}\right)}$$

The lumped uncertainties $\Delta(x,t)$ includes uncertain parameters $\Delta f(x,t)$ and the external disturbance $d(t)$ that has the equation as below

$$\Delta(x,t) = 0.01\sin 10x_1 + 0.05\cos x_2 + 0.25\sin t + \text{rand}(1) \quad (33)$$

The desired angle of the inverted pendulum is given as below

$$x_d(t) = 1 + e^{-t} - 0.25e^{-4t} \quad (34)$$

The parameters of the inverted pendulum on cart are used for the simulation: $m_c = 1(kg)$; $m_p = 0.1(kg)$; $2l = 0.5(m)$; $g = 9.81(m/s^2)$. The parameters of controllers: $\alpha_1 = 3$, $\alpha_2 = 2$, $p = 3$, $q = 2$, $k_1 = 5$, $k_2 = 4$, $\gamma_1 = 0.8$, $\gamma_2 = 1.3$. The initial values: $\theta(0) = 0.3$, and $\dot{\theta}(0) = -0.2$

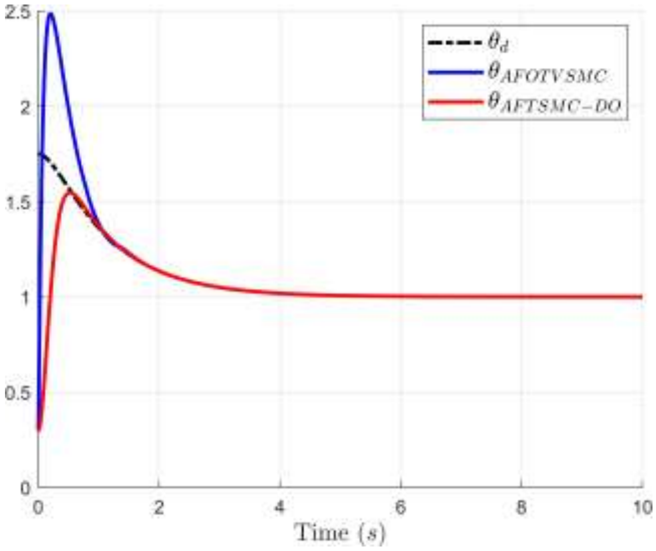

Fig. 2. The angle of the inverted pendulum $\theta(t)$.

The simulation was conducted using an inverted pendulum on a cart to evaluate the performance of the proposed AFTSMC-DO. As illustrated in Figs. 2 and 3, the angular position $\theta(t)$ and angular velocity $\dot{\theta}(t)$ under AFTSMC-DO exhibit faster convergence and reduced steady-state error compared to AFOTVSMC. Fig. 4 shows that the sliding variable associated with AFTSMC-DO converges to zero more rapidly, indicating faster error dynamics. Furthermore, the control input generated by AFOTVSMC (Fig. 5) is significantly more aggressive than that of AFTSMC-DO, which confirms that the proposed controller achieves robust performance with reduced control effort. Lastly, as shown in Fig. 6, the disturbance observer in AFTSMC-DO accurately estimates the lumped uncertainty in real time, demonstrating high estimation precision and enhanced disturbance rejection.

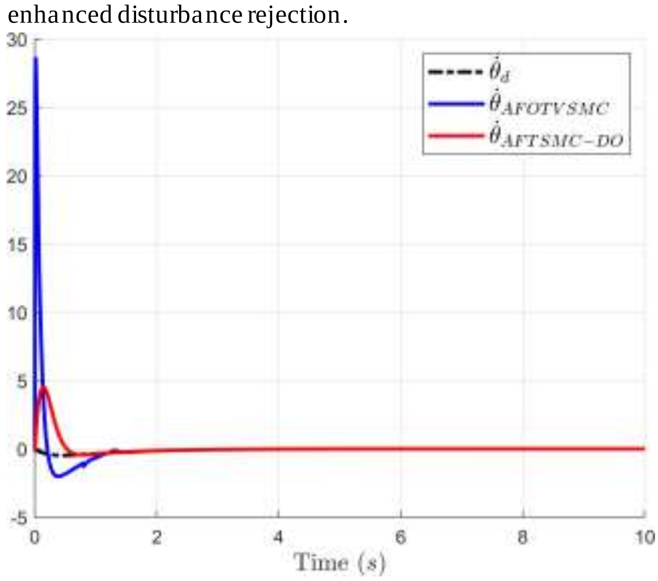

Fig. 3. The angular velocity of the inverted pendulum $\dot{\theta}(t)$.

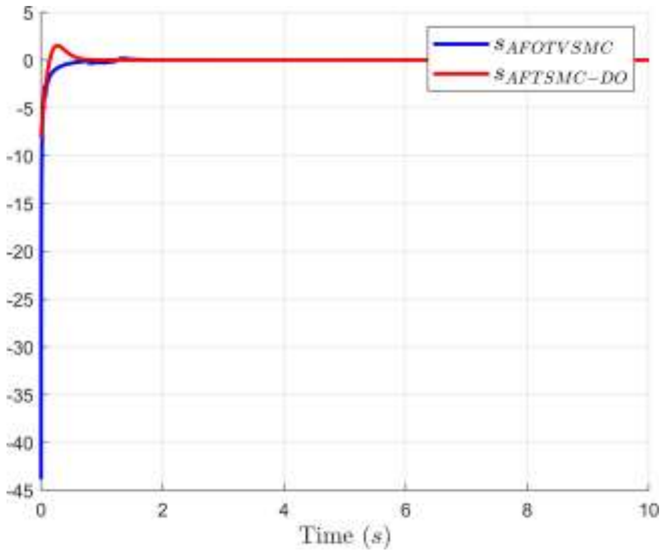

Fig. 4. The sliding surface $s(t)$.

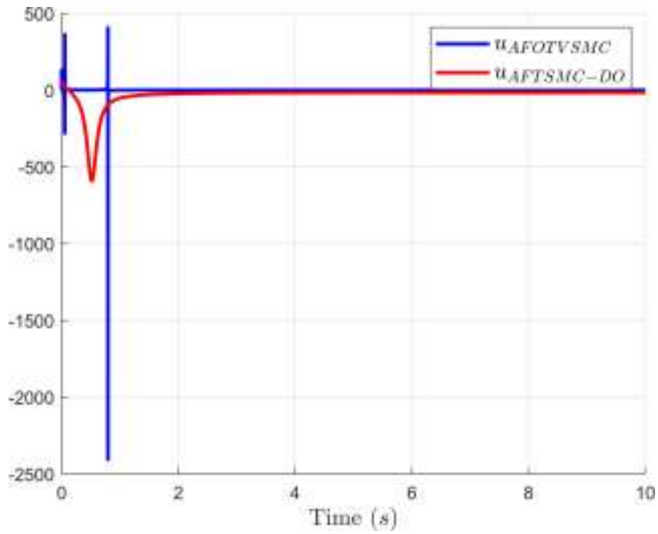

Fig. 5. The control law $u(t)$.

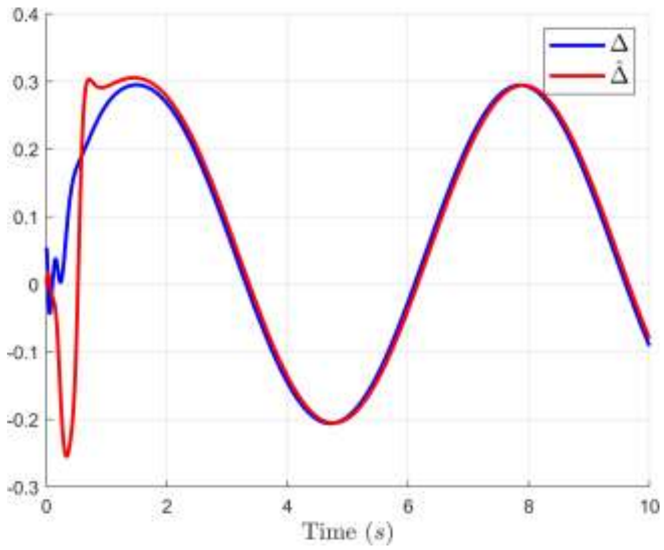

Fig. 6. The disturbance estimation $\hat{\Delta}(x,t)$.

V. CONCLUSIONS

This paper presented a novel Adaptive Fixed-Time Terminal Sliding Mode Control scheme augmented by a Disturbance Observer (AFTSMC-DO) for nonlinear second-order systems under bounded matched uncertainties. By incorporating a low-pass filtered observer into the control loop and using a BLF-based ensures fixed-time convergence of both tracking and disturbance estimation errors.

Through rigorous stability analysis and simulation on an inverted pendulum, AFTSMC-DO demonstrated significantly improved transient performance, robustness, and lower control effort compared to AFOTVSMC. These results suggest that the proposed controller is well-suited for safety-critical and real-time control systems. Future work will focus on experimental validation and extension to multi-input multi-output (MIMO) systems and networked control frameworks.

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Hoang Duc Long was born in Hanoi, Vietnam. He received the B.Sc. degree in mechatronics and the M.S. degree in automation from the Faculty of Control Engineering, Le Quy Don Technical University, Hanoi, Vietnam, in 2012 and 2020, respectively. He received a Ph.D. degree in control in technical systems from the Faculty of Control Systems and Robotics, Saint Petersburg State University of Information Technologies, Mechanics and Optics (**ITMO University**), Saint Petersburg, Russia, in 2025.

He is currently a lecturer at the Department of Automation and Computing Techniques in the Faculty of Control Engineering, Le Quy Don Technical University, Hanoi, Vietnam. His research interests include dynamics of control systems and robotics, uncertain nonlinear systems, mathematical modeling and analysis of systems, and stability of dynamic systems.

Dr. Hoang has had more than 20 publications in the journals of Scopus and Web of Science.